Quantifying counterparty risk

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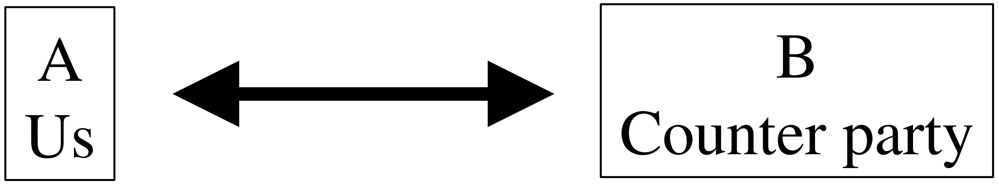
Agenda

- Definitions of counterparty risk
- One sided counterparty risk
- Product specific considerations
- One or two sided counterparty risk?
- Model requirements
- Trading of counterparty risk
- Numerical implementations
  - 3 suggestions, “regression in MC” in detail
  - How, what to look out for, live demo, …
- Portfolio calculations by aggregation
- Conclusion

Counterparty risk definition

- The risk of losing money on a portfolio of derivative contracts when a counterparty default



- Cashflows at default time τ before maturity T:
  - Payments before τ: according to the contract
  - At default of counterparty B:
    - NPV>0: counterparty owes us money and pays RR^{B}* NPV to us
    - NPV<0: we owe the counterparty money and pay them in full
  - At our default A:
    - NPV>0: counterparty owes us money and pay in full
    - NPV<0: we owe the counterparty money and pay RR^{A}*NPV

Purpose of measuring counterparty risk

- Reservations for future exposure
  - Lines control
- Pricing
  - Special price for each counterparty
- Hedging
- Related, but NOT considered here:
  - VaR, expected shortfall
    - Typical 10 trading days
  - Economic Capital
    - 99.7% quantile of unexpected losses on 1y horizon
- Accuracy needed
  - Cash-flows/exposure on individual days or the big picture?

Other means of managing counterparty risk

- **Netting agreements**
  - Net between contracts with the same counterparty, also across asset classes
  - Almost always in place
- **Collateral agreements**
  - Make sure exposure never exceeds a given threshold by securing the position with collateral
  - Typical for interbank counterparties and large clients
- **Early termination clauses**
- **Corporate counterparties**
  - Smaller portfolios, but no collateral and higher credit risk

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Counterparty risk math definition

$$NPV(\tau) = E_{\tau}\left[CF(\tau, T)\right], \text{ seen from us, counterparty A}$$

$$\text{payoff}^{D}(t) = 1_{\tau > T} CF(t,T) + 1_{t < \tau \le T}\left[CF(t,\tau) + df(t,\tau)\, NPV(\tau)(\gamma^{A} + \gamma^{B})\right]$$

$$\gamma^{A} = 1_{\tau = \tau^{A}}\left(RR^{A} 1_{NPV(\tau)<0} + 1_{NPV(\tau)>0}\right), \text{ A defaults}$$

$$\gamma^{B} = 1_{\tau = \tau^{B}}\left(RR^{B} 1_{NPV(\tau)>0} + 1_{NPV(\tau)<0}\right), \text{ B defaults}$$

- This is two sided counterparty risk, both parties can default
- One sided: put $\gamma^{A}=0$ (we cannot default)

One sided counterparty risk

- $\gamma^A=0$, we only consider defaults of our counterparty
- With a bit of tedious, but simple, algebra and law of iterated expectations:

$$E_t(\text{payoff}^D(t)) = E_t(\text{payoff}(t)) - (1-RR^B)E_t\left[1_{t<\tau\leq T}\, df(t,\tau)\, NPV^+(\tau)\right]$$

Value without counterparty risk

Option part in default case
Call 0-strike

- RR assumed deterministic
- Adds level of optionality: we need (a function of) the value at a future default date
- Mean over τ and NPV values

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Products

- Bank loan portfolio
  - Simple --- value of underlying do not change much!
  - Might have extension clause, correlated to credit quality, complicates matters!
- IRS
  - Simple
  - Value 0 at initiation, but value $\neq 0$ at future dates
  - Fast approximations can be made
- FX
- Swaptions
  - Cash/physical settled makes difference wrt. final maturity
  - Option on option, stochastic volatility
- Credit products
  - Take correlation between underlying and counterparty into account
- Equity
- Portfolios of the full monty…

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IRS: Interest Rate Swaps

- **The general expression simplifies:**

$$\mathrm{IRS}^{D}(t) = \mathrm{IRS}(t) - (1 - RR^{B}) \int_{t}^{T} \mathrm{swaption}(t, s, T, K)\, dQ(\tau \leq s)$$

- **Q describe default times by hazard rates from CDS quotes**
  - **CDS up to 10y, trades up to 30y**
- **Independence between τ and rates assumed**
  - **Rate distribution does not depend on τ, i.e. we get vanilla swaption**
- **Weighting options with default probabilities**

Impact on price on a single IRS

- IRS^D quote: coupon that gives $IRS^D=0$
- Market data as of 21-MAR-2007 (rates, vol)
- CDS scenarios:

	Survival Prob		
Tenor	Low CDS 5y=30bp	Medium CDS 5y=100bp	High CDS 5y=300bp
5y	97.50%	91.92%	77.67%
10y	95.07%	84.50%	60.35%
15y	92.71%	77.69%	46.89%
20y	90.40%	71.42%	36.43%

- Results:

			Diff in rates in bp		
Tenor	Maturity Date	Rate	Low CDS 5y=30bp	Medium CDS 5y=100bp	High CDS 5y=300bp
5y	Fri-23-Mar-2012	4.1230%	0.17	0.53	1.50
10y	Thu-23-Mar-2017	4.1890%	0.50	1.62	4.44
15y	Wed-23-Mar-2022	4.2850%	0.91	2.87	7.55
20y	Tue-23-Mar-2027	4.3290%	1.25	3.93	9.96

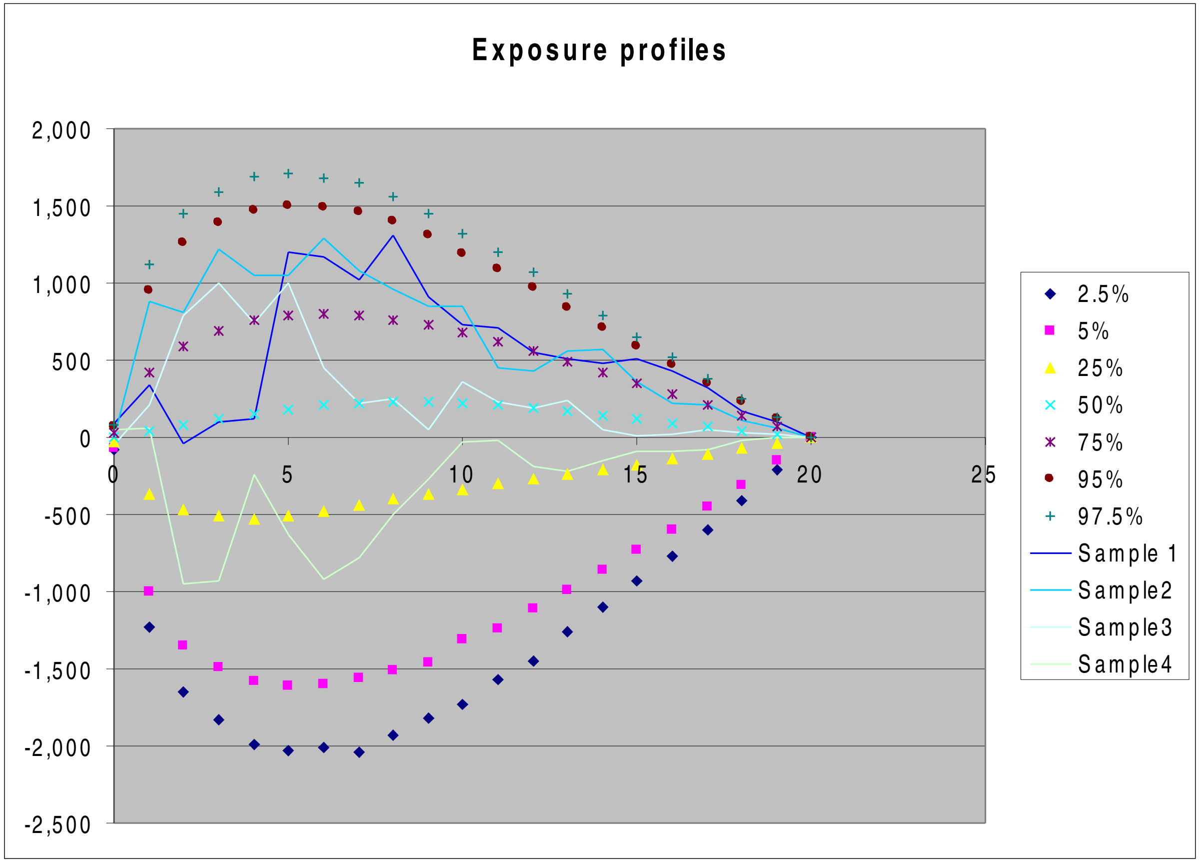
- Adjustments a bit (times ½) lower than in Brigo & Masetti (2004)
  - Vol assumptions different, …

One or two sided counterparty risk?

- **Seen from our point of view:**
  - One sided counterparty risk is enough
- **But the counterparty has the same view**
  - So two sided counterparty risk seem to be the way to go if parties should agree on a common price
- **Value depends mostly on difference in CDS spreads**
  - As an approximation only see it from the highest rated counterparty's side

Exposure profiles

- **Jumps at payments dates**
- **Need to calculate option on full portfolio**
  - Cannot do it trade by trade due to netting
  - Exposures occur at different dates for different swaps
- **Single trade/portfolio numbers**
  - Quantiles, max, quantiles of max, averaging, etc.



Portfolios of interest rate swaps

- Netting of positions & exposure
  - Simple example: payer and receiver swap with same strike and maturity
- "Swaption" on general cashflow of (libor) payments
- Damiano Brigo & Massimo Masetti, 2005 find approximate equations
  - Either strictly payer or receiver portfolios
  - Both payer and receiver portfolios give complications
    - This will usually be the case!

- This is going in the direction of specializing for specific products/type of positions/…
- In general assuming little about the products or portfolio composition, then more general models must be used…

General or specific models: I would say general!

- Even with specific models there is a limit to what can be handled
  - Realistic swap portfolio
- For homogeneous portfolios
  - Simple regression techniques will be sufficient in order to give good overview
  - Might be rather add-hoc, but never the less be sufficient
  - Per trade: current NPV + add-on
  - Add-on depends on currency (vol?), time to maturity, counterparty rating
  - Give discount in add-on in order to take typical netting into account
- For non-homogeneous portfolios
  - Something more general needs to be done anyway
  - In particular for exotics

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Model requirements

- In general: adds level of optionality
  - Needs value at a future date τ of future remaining payments
- NPV can depend on history up to default
  - Simple example: physical settled swaption past expiry date, ITM/OTM?
- Options
  - Before expiry: needs to price an option on an option
  - SV models
- Correlation between default time and underlying
  - Independence might be reasonable for rates/defaults
  - Credit/equity products: correlation between reference name and counterparty needs to be taken into account
- The interest is in calculating the option part in the adjusted price
  - Might use other models than the pricing model as the focus is different

Trading of counterparty risk

- So far: pricing taking counterparty risk into account
  - Used as MTM (seldom) or only in lines surveillance
- Hedging counterparty risk
  - Swap, option desks, etc. hedge counterparty risk with credit desk in order to trade more with a given limit
  - Jump To Default risk, $(1-RR^B)NPV^+$, current exposure
  - Hazard risk: potential future exposure
- Make counterparty risk a market risk like delta/vega/…
- Difficult to do for smaller names with illiquid CDS market
- Risk number calculation adds a lot to numerical problems
  - Would requite a lot more simulations than just the pricing of counterparty risk

Risk neutral measure $\leftrightarrow$ real world measure

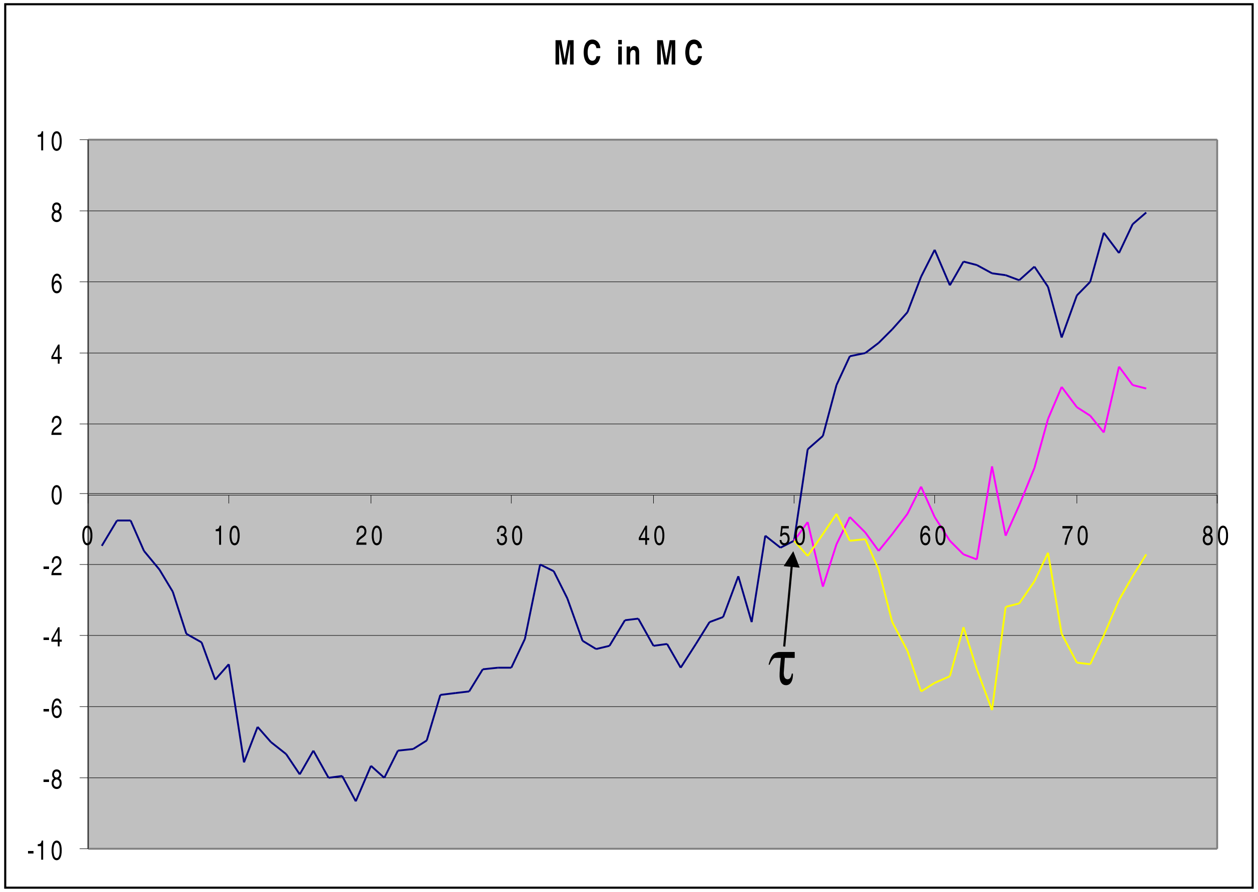
- **Risk neutral measure:**
  - What we have worked with so far
  - Used for pricing and hedging
- **Real world measure:**
  - Risk management might argue that this is more relevant for lines, reservations, etc.
  - Both for market factors and default risk
  - Different models

Numerical implementation: MC on Grid

- Original idea by Jesper Andreasen
- Suitable when both Grid and MC models available
  - And products can be priced in grid
- Do grid once backwards
  - Store value for every grid point
- Simulate MC state variables AND defaults forward
  - Pick a grid box based on default time and state
    - The value of future payments are pre computed from the grid!
  - Allows for default/state variable correlation
- Haven't tried it….
- Another idea: Do grid for default state as well, increases dimensionality, but only 2 states in new direction

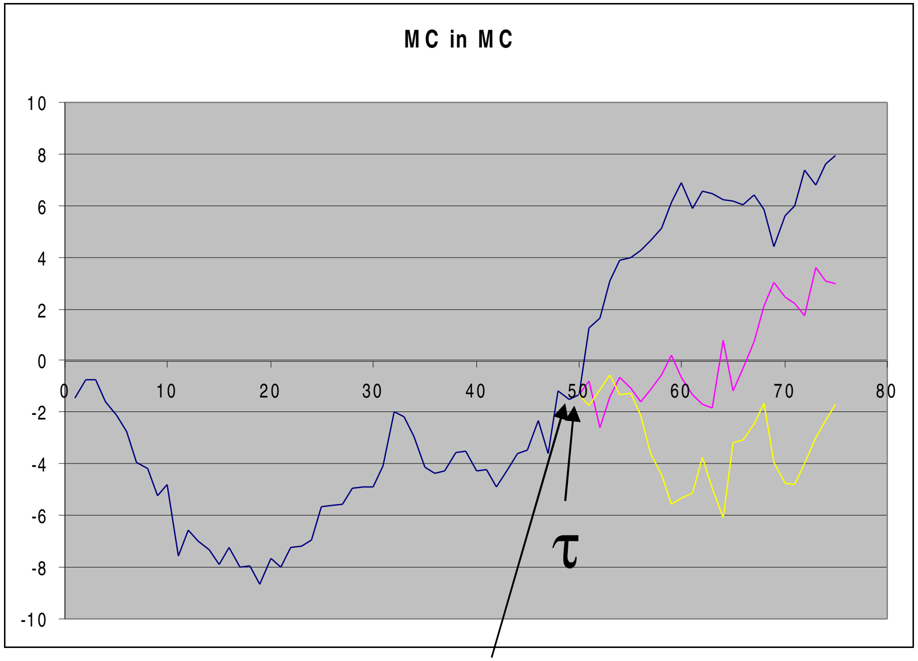
Numerical implementation: MC in MC

- **Procedure:**
  - Simulate τ
  - Value future CF by MC from that point
- **Optimizations**
  - Product dependent
  - Path in time
  - Jump to date
- **Cross asset portfolios/hybrids/…**
  - Huge MC engine
- **Most exotics are in MC models these days…**
- **MC in MC explodes computationally, #sim^2**



Numerical implementation: Regression in MC

- **Procedure:**
  - Simulate τ
  - Value future CF by regression at τ
- **Like Longstaff-Schwartz regression for early exercise boundary**
- **Feasible computationally:**
  - 2 x #sim (or less)
  - Perhaps already doing the sim for early exercise boundary



Do regression instead of MC

$$E_t(\text{payoff}^D(t)) = E_t(\text{payoff}(t)) - (1 - RR^B)\, E_t\left[1_{t<\tau\le T}\, df(t,\tau)\, NPV^+(\tau)\right]$$

Regression in MC

- **At each time t, predict value of future cashflow by regression:**

$$NPV(\tau) = \alpha(\tau)' x(\tau) + \varepsilon = \sum_k \alpha_k(\tau) x_k(\tau) + \varepsilon$$

- **NPV(τ): value of future cashflows at time τ, see next slide on how to get**
  - **Note: NOT $NPV^+(\tau)$, as this would make the regression fit worse.**
  - **Take positive part after the regression!**
- **$\alpha(\tau)$: linear regression coefficients at time τ**
- **x(τ): regression variables like libor, swaprate, $swaprate^2$, etc.**
  - **Choose with care!**
  - **Should predict value by just knowing current state of the world**
- **$\varepsilon(\tau)$: “noise” vector**

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Regression in MC procedure

- Make pre simulation
  - Store a set of full paths
  - Evaluate forward in time as usual, store values for each time step
  - Now go backwards in time in order to find value of future CF at each time
  - Find regression coefficients from regression variabes
- Make simulation in model:
  - Simulate defaults times, either given externally from “credit model”, or given by the model itself when correlation between default and asset needed.
  - Simulate underlyings, rate, etc., as usual
  - Evaluate at time t forward in time as usual, but for counter party risk:
    - Return 0 if not defaulted, i.e. $t<\tau$
    - $(\alpha' x)(t)$ if defaulted here, i.e. $t=\tau$
    - Pass on current value if previously defaulted, $t>\tau$ (can in some cases be disregarded)

Regression in MC, considerations

- Regression variables:
  - Should predict value of remaining cash-flow from current state of the world
  - Can be a bit tricky to find the best
    - Experiment!
    - Both short end and long end of curve
    - Value of vol with SV models
    - Use powers of variables
  - Need more experience for exotic stuff
- Regress on full range of values instead of a lot of zeros and the positive part, i.e. $NPV(\tau)$ instead of $NPV^{+}(\tau)$.
  - Better fit at fitting stage
  - Better prediction at prediction state
  - Makes aggregation across trades possible at a later stage!

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Live demo…..

- Implemented as “aggregate model”:
  - All models can interact with the default model (if they adhere to the interface!)
  - If correlation default ↔ asset needed the model can provide default times itself.
- Implemented with new keyword in trade description to get regression variable
  - Means pricing and counterparty risk can be done simultaneously!
- Still lots of rough edges! Work in progress!
- This stuff actually works ☺
  - Give values in line with “closed form” solution for swaps
  - Reasonable performance
  - Low overhead compared to usual pricing (at least for exotics…)

Future directions

- **Implement risk**
  - Should be an easy extension
  - Credit risk part by standard trick of swapping differentiation and MC mean (integration)
- **Implement the counterparty interface on all models**
- **Implement plumming to value a whole portfolio of trades in one go**
  - "Super model" to value all assets
  - Might NOT be needed if the same defaults τ are used in all models and models return NPV(τ) as a vector for all default times.
  - Possible to aggregate information from several independent trades/models

$$NPV^{+}_{total}(\tau) = \left[\sum_{\text{trades}} NPV_{trade}(\tau) \right]^{+}$$

  - More accurate regression because tailored to each individual trade
  - Simple to aggregate. Store values from, say, EOD, so effect of new trades can easily be calculated

Conclusion

- Counterparty risk adds level of optionality
- Netting agreements $\rightarrow$ we should look at a portfolio level
  - Might be distributed across books at different trading desks
  - A challenge to infrastructure and systems
- Need to decide on strategy
  - Get efficient approximations for simple single asset class/product portfolios
  - Do all products/asset classes together in huge MC engine
  - Some route in between or combination…
  - Computations could be challenging!
- Pricing of counterparty risk can be obtained in roughly the same time as an MC price.
  - Good enough as probably most interesting for exotics anyway

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