Quantifying counterparty risk

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Agenda

- **Definitions of counterparty risk**
- **One sided counterparty risk**
- **Product specific considerations**
  - **Interest rate swaps**
- **One or two sided counterparty risk?**
- **Model requirements**
- **Numerical implementations**
- **Trading of counterparty risk**
- **Conclusion**

Counterparty risk definition

- **The risk of losing money on a portfolio of derivative contracts when a counterparty defaults**
- **Cashflows at default time τ before maturity T:**
  - **Payments before τ: according to the contract**
  - **At default of counterparty B:**
    - **NPV>0: counterparty owes us money and pays RR^B* NPV to us**
    - **NPV<0: we owe the counterparty money and pay them in full**
  - **At our default A:**
    - **NPV>0: counterparty owes us money and pay in full**
    - **NPV<0: we owe the counterparty money and pay RR^A*NPV**

Purpose of measuring counterparty risk

- Reservations for future exposure
  - Lines control
- Pricing
  - Special price for each counterparty
- Hedging
- Related, but NOT considered here:
  - VaR, expected shortfall
    - Typical 10 trading days
  - Economic Capital
    - 99.7% quantile of unexpected losses on 1y horizon

Other means of managing counterparty risk

- **Netting agreements**
  - **Net between contracts with the same counterparty, also across asset classes**
  - **Almost always in place**
- **Collateral agreements**
  - **Make sure exposure never exceeds a given threshold by securing the position with collateral**
  - **Typical for interbank counterparties and large clients**
- **Early termination clauses**
- **Corporate counterparties**
  - **Smaller portfolios, but no collateral and higher credit risk**

Counterparty risk math definition

$$NPV(\tau) = E_\tau\left[CF(\tau, T)\right]$$

$$\text{payoff}^{\text{D}}(t) = 1_{\tau > T}\, CF(t,T) + 1_{t < \tau \le T}\left[CF(t,\tau) + df(t,\tau)\, NPV(\tau)(\gamma^A + \gamma^B)\right]$$
$$\gamma^A = 1_{\tau = \tau^A}\left(RR^A 1_{NPV(\tau)<0} + 1_{NPV(\tau)>0}\right)$$
$$\gamma^B = 1_{\tau = \tau^B}\left(RR^B 1_{NPV(\tau)>0} + 1_{NPV(\tau)<0}\right)$$

- This is two sided counterparty risk, both parties can default
- One sided: put $\gamma^A=0$ (we cannot default)

One sided counterparty risk

- $\gamma^A=0$, we only consider defaults of our counterparty
- With a bit of tedious, but simple, algebra and law of iterated expectations:

$$E_t(\text{payoff}^{D}(t)) = E_t(\text{payoff}(t)) - (1 - RR^B)E_t\left[1_{t<\tau\leq T}\, df(t,\tau) NPV^+(\tau)\right]$$

Value without counterparty risk

Option part in default case
Call 0-strike

- RR assumed deterministic
- Adds level of optionality: we need (a function of) the value at a future default date
- Mean over τ and NPV values

Products

- Bank loan portfolio
  - Simple --- value of underlying do not change much!
- IRS
  - Simple
  - Value 0 at initiation, but value $\neq$ 0 at future dates
  - Fast approximations can be made
- FX
- Swaptions
  - Cash/physical settled makes difference wrt. final maturity
  - Option on option, stochastic volatility
- Credit products
  - Take correlation between underlying and counterparty into account
- Equity
- Portfolios of the full monty…

IRS: Interest Rate Swaps

- **The general expression simplifies:**

$$\mathrm{IRS}^D(t) = \mathrm{IRS}(t) - (1 - RR^B)\int_t^T \mathrm{swaption}(t, s, T, K)\, dQ(\tau \le s)$$

- **Q describe default times by hazard rates from CDS quotes**
  - **CDS up to 10y, trades up to 30y**
- **Independence between τ and rates assumed**
  - **Rate distribution does not depend on τ, i.e. we get vanilla swaption**
- **Weighting options with default probabilities**

Impact on price on a single IRS

- IRS^D quote: coupon that gives $IRS^D=0$
- Market data as of 21-MAR-2007 (rates, vol)
- CDS scenarios:

Tenor	Survival Prob Low CDS 5y=30bp	Medium CDS 5y=100bp	High CDS 5y=300bp
5y	97.50%	91.92%	77.67%
10y	95.07%	84.50%	60.35%
15y	92.71%	77.69%	46.89%
20y	90.40%	71.42%	36.43%

- Results:

Tenor	Maturity Date	Rate	Diff in rates in bp Low CDS 5y=30bp	Medium CDS 5y=100bp	High CDS 5y=300bp
5y	Fri-23-Mar-2012	4.1230%	0.17	0.53	1.50
10y	Thu-23-Mar-2017	4.1890%	0.50	1.62	4.44
15y	Wed-23-Mar-2022	4.2850%	0.91	2.87	7.55
20y	Tue-23-Mar-2027	4.3290%	1.25	3.93	9.96

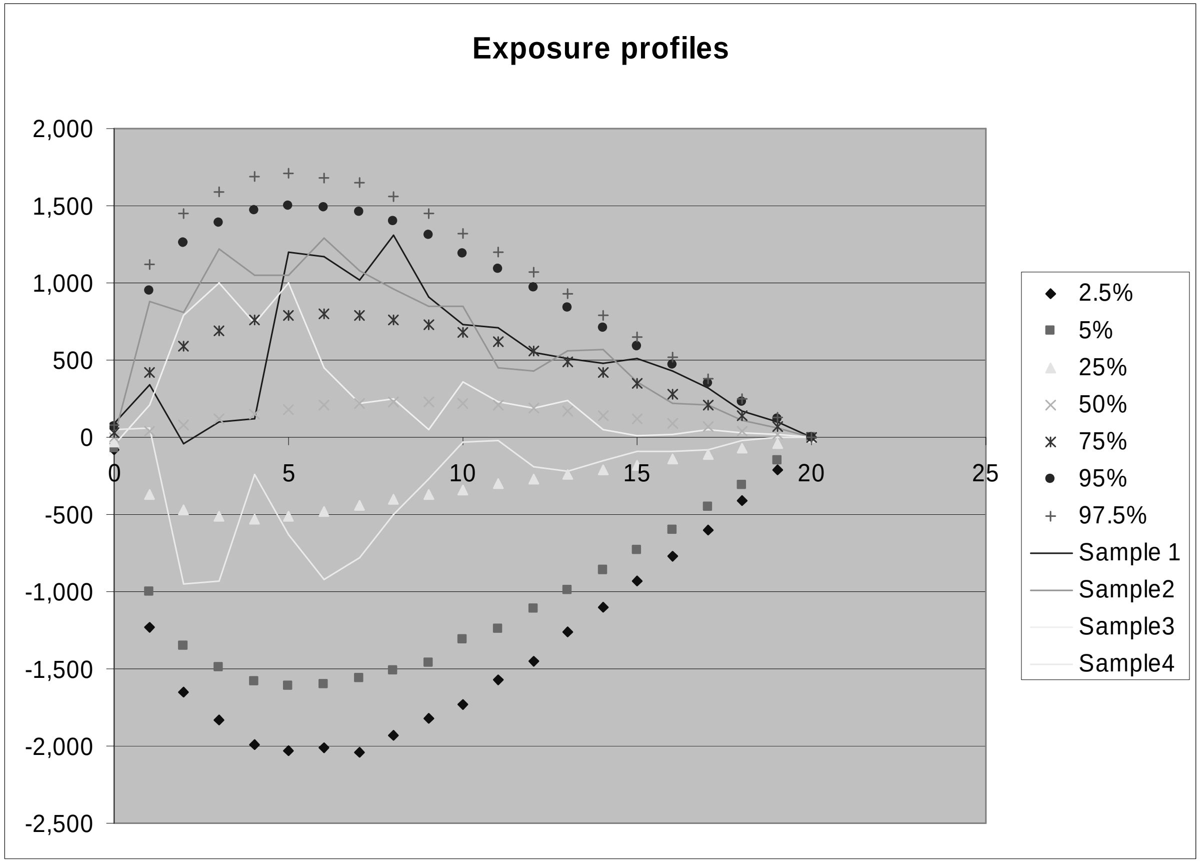
- Adjustments a bit (times ½) lower than in Brigo & Masetti (2004)
  - Vol assumptions different, …

One or two sided counterparty risk?

- Seen from our point of view:
  - One sided counterparty risk is enough
- But the counterparty has the same view
  - So two sided counterparty risk seem to be the way to go if parties should agree on a common price
- Value depends mostly on difference in CDS spreads
  - As an approximation only see it from the highest rated counterparty’s side

Exposure profiles

- **Jumps at payments dates**
- **Need to calculate option on full portfolio**
  - **Cannot do it trade by trade due to netting**
  - **Exposures occur at different dates for different swaps**
- **Single trade/portfolio numbers**
  - **Quantiles, max, quantiles of max, averaging, etc.**



Portfolios of interest rate swaps

- **Netting of positions & exposure**
  - **Simple example: payer and receiver swap with same strike and maturity**
- **“Swaption” on general cashflow of (libor) payments**
- **Damiano Brigo & Massimo Masetti, 2005 find approximate equations**
  - **Either strictly payer or receiver portfolios**
  - **Both payer and receiver portfolios give complications**
    - **This will usually be the case!**

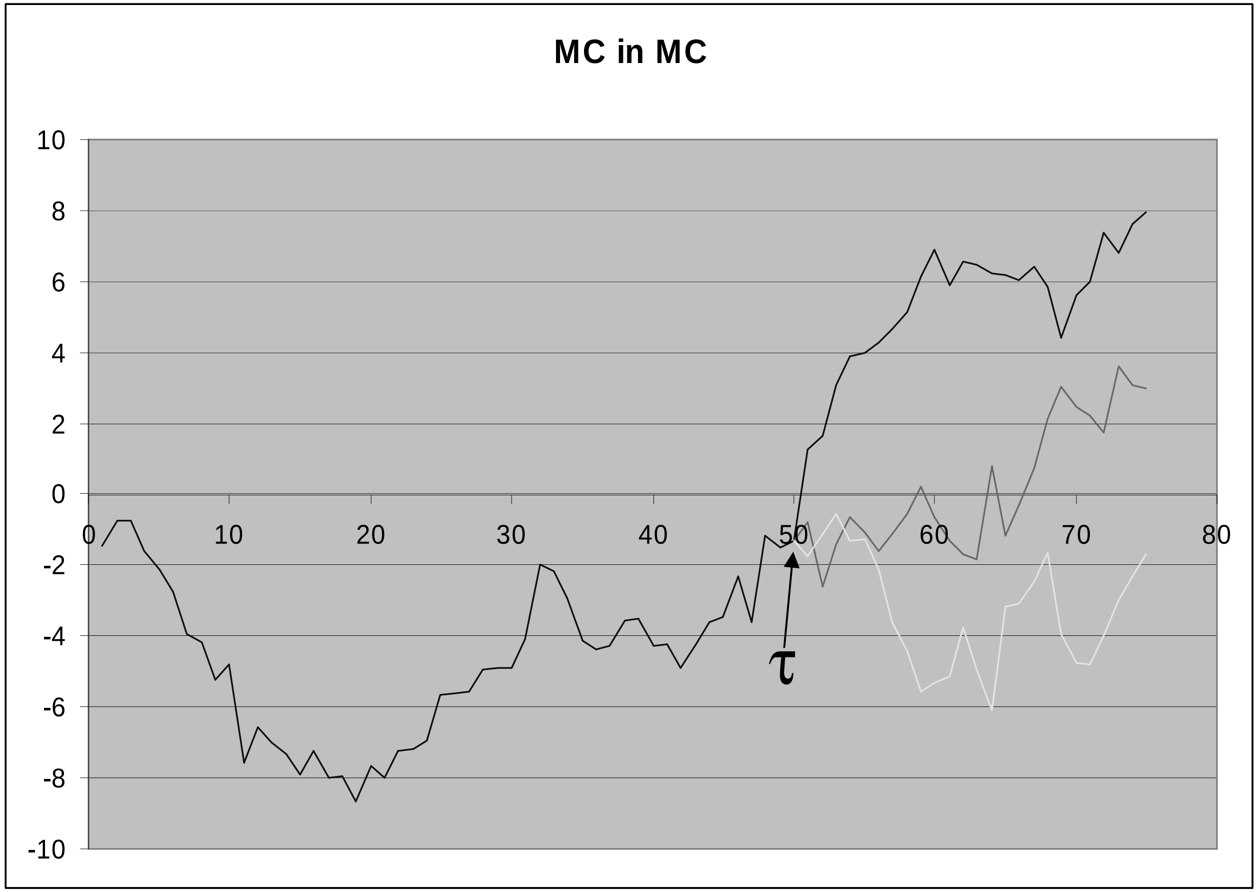
- **This is going in the direction of specializing for specific products/type of positions/…**
- **In general assuming little about the products or portfolio composition, then more general models must be used…**

Model requirements

- In general: adds level of optionality
  - Needs value at a future date τ of future remaining payments
- NPV can depend on history up to default
  - Simple example: physical settled swaption past expiry date, ITM/OTM?
- Options
  - Before expiry: needs to price an option on an option
  - SV models
- Correlation between default time and underlying
  - Independence might be reasonable for rates/defaults
  - Credit/equity products: correlation between reference name and counterparty needs to be taken into account
- The interest is in calculating the option part in the adjusted price
  - Might use other models than the pricing model as the focus is different

Numerical implementation: MC in MC

- **Procedure:**
  - **Simulate τ**
  - **Value future CF by MC from that point**
- **Optimizations**
  - **Product dependent**
  - **Path in time**
  - **Jump to date**
- **Cross asset portfolios/hybrids/ …**
  - **Huge MC engine**



Numerical implementation: MC on Grid

- Original idea by Jesper Andreasen
- Suitable when both Grid and MC models available
  - And products can be priced in grid
- Do grid once backwards
  - Store value for every grid point
- Simulate MC state variables AND defaults forward
  - Pick a grid box based on default time and state
    - The value of future payments are pre computed from the grid!
  - Allows for default/state variable correlation
- Haven’t tried it yet….
- Another idea: Do grid for default state as well, increases dimensionality, but only 2 states

Trading of counterparty risk

- So far: pricing taking counterparty risk into account
  - Used as MTM (seldom) or only in lines surveillance
- Hedging counterparty risk
  - Swap, option desks, etc. hedge counterparty risk with credit desk in order to trade more with a given limit
  - Jump To Default risk, $(1-RR^B)NPV^+$, current exposure
  - Hazard risk: potential future exposure
- Make counterparty risk a market risk like delta/vega/…
- Difficult to do for smaller names with illiquid CDS market
- Risk number calculation adds a lot to numerical problems
  - Would requite a lot more simulations than just the pricing of counterparty risk

Risk neutral measure $\leftrightarrow$ real world measure

- **Risk neutral measure:**
  - What we have worked with so far
  - Used for pricing and hedging
- **Real world measure:**
  - Some would argue that this is more relevant for lines, reservations, etc.
  - Both for market factors and default risk
  - Different models

Conclusion

- Counterparty risk adds level of optionality
- Netting agreements → we should look at a portfolio level
  - Might be distributed across books at different trading desks
  - A challenge to infrastructure and systems
- Need to decide on strategy
  - Get efficient approximations for simple single asset class/product portfolios
  - Do all products/asset classes together in huge MC engine
  - Some route in between…
  - Computations are going to challenging!

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