Nordea

CDO modelling from a practitioner’s point of view: What are the real problems?

Jens Lund
jens.lund@nordea.com
7 March 2007

Bridging between academia and practice

- **The speaker**
- **iTraxx, standard CDOs and conventions**
- **Gaussian copula model**
  - **CDO behaviour**
  - **Correlation smile**
  - **Compound $\leftrightarrow$ base correlations**
  - **Some base correlation issues**
- **What is a good model?**
  - **Interpolation and extrapolation, non standard tranches/portfolios**
  - **Market information**
  - **Hedge ratios**
- **Implementation considerations**
  - **MC strategies, how to simulate**
  - **Risk numbers for all market data**
  - **Fast recursive techniques, conditional independence**
  - **Other model proposals**
- **What makes a good pratitioner?**
- **Conclusion**

Jens Lund

- **Head of Product Development, Nordea Markets**
- **Background:**
  - **Nov 1996: M.Sc. in statistics, University of Copenhagen**
  - **Feb 2000: Ph.D. in statistics, The Royal Veterinary and Agricultural University**
  - **Mar 2000 onwards: with Nordea, Product Development**
  - **Has done a lot of the credit modelling work in Nordea**
- **Team:**
  - **5 members, various degrees of experience, mainly Ph.D. in natural science, looking for more people**
  - **2 associated programmers helping with interface to trading system**
  - **Responsible for all derivatives modelling and calculations (NPV, risk,…)**
  - **Scripting language for description of all derivatives**
  - **Interest rates, credit derivatives, inflation, equity, …**

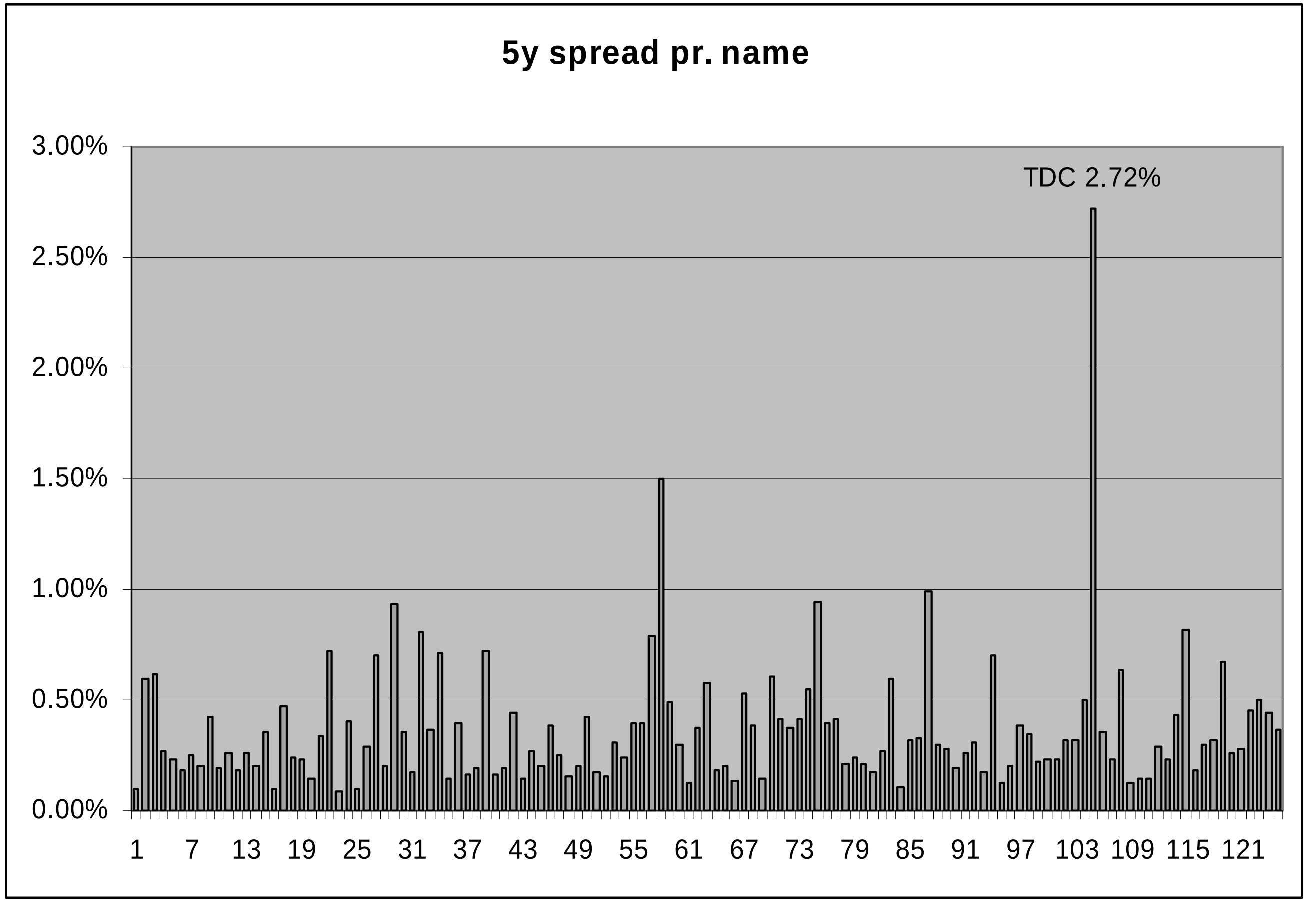
iTraxx standard portfolio/CDS

- **iTraxx Europe**
  - 125 liquid names
  - Underlying index CDSes for sectors
  - 5Y, 7Y & 10Y maturity
  - 5 standard CDO tranches, first to default baskets, options
  - US index CDX
- **3m, act/360, last 20 date roll, CDS pay accrued fee**
- **Index composition adjusted every 6m**
- **Index CDS trades at a fixed spread with accrued fee ---**

  Traded with upfront premium (but quoted on spread)
  - Together with last 20 date roll this ensures liquidity and (minus counterparty risk) perfect netting of trades.

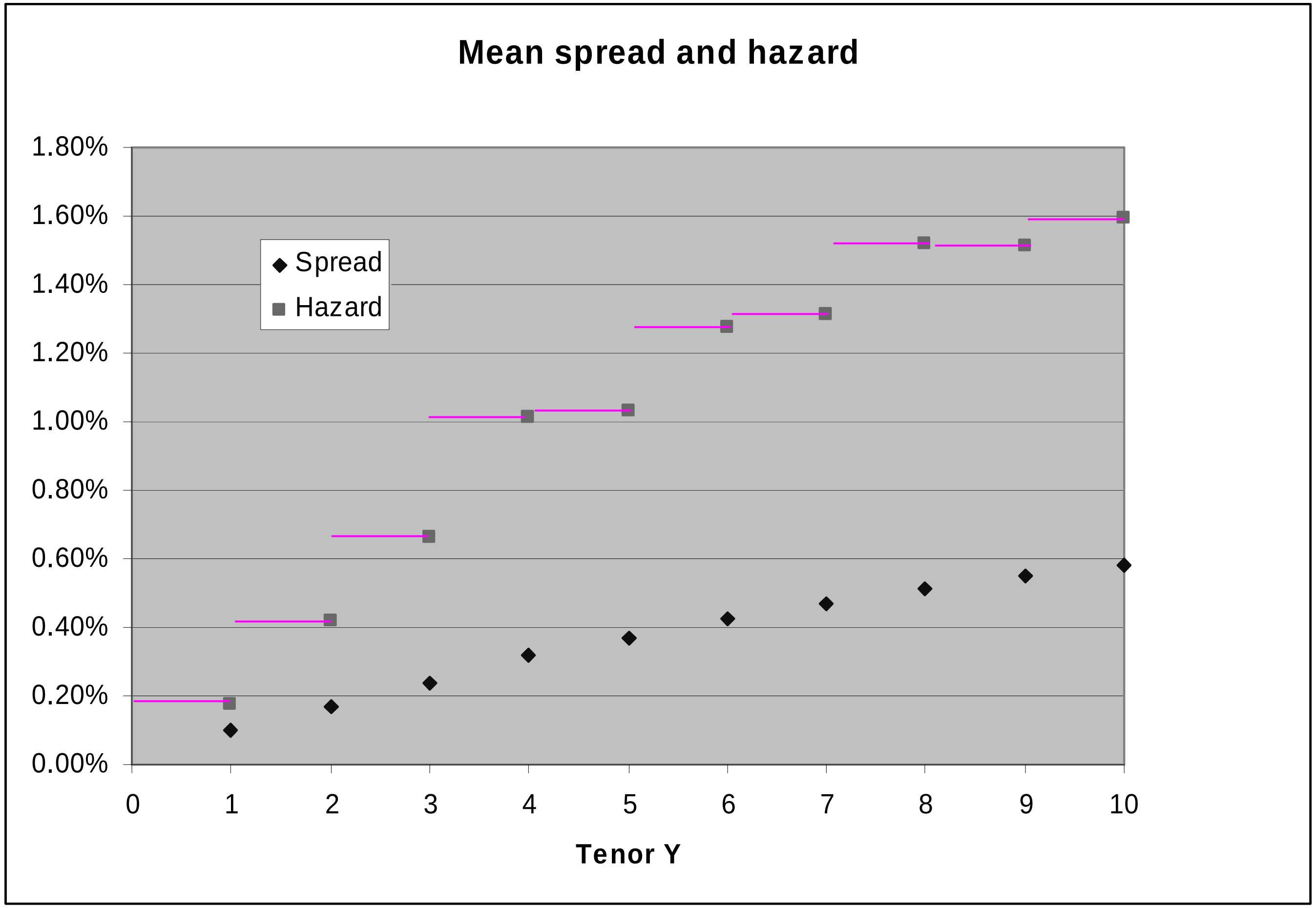
Nordea

iTraxx, distribution of 5y spreads

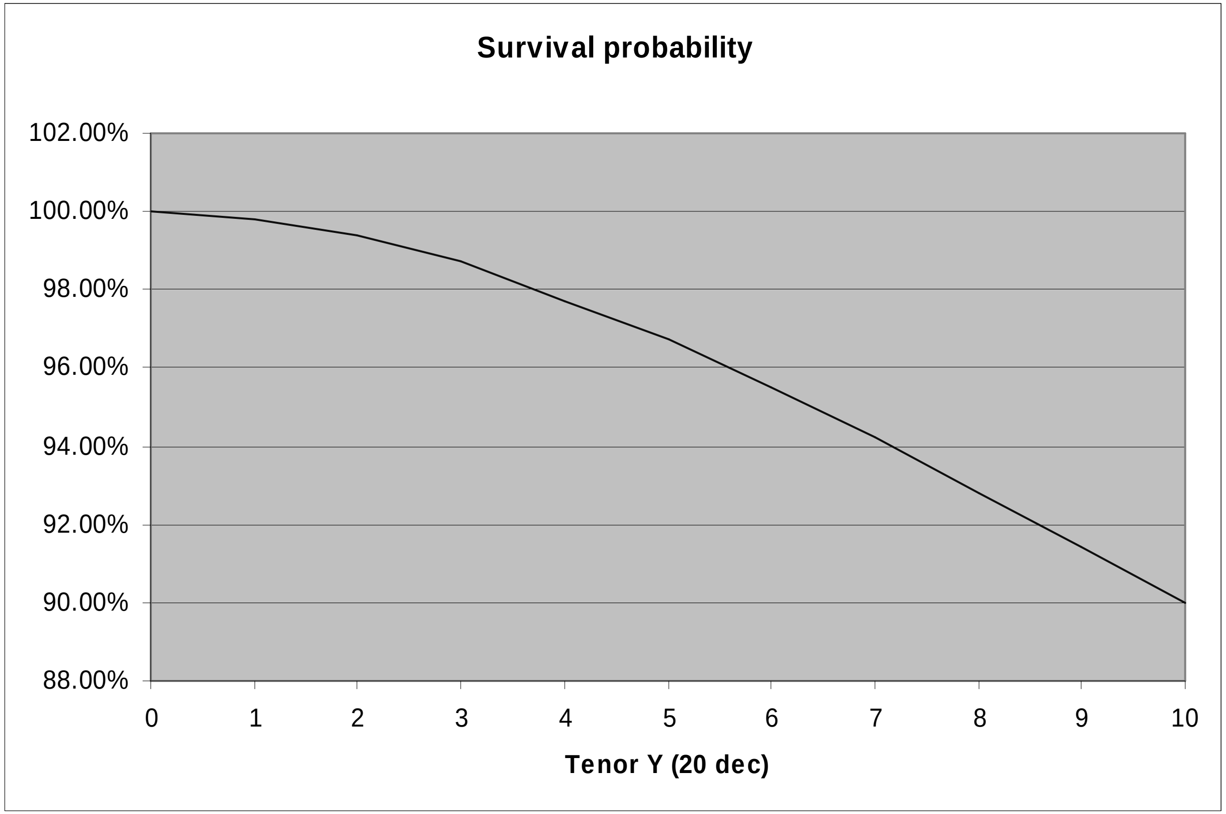


TDC 2 Nov 2005: 293bp -> 268bp -> 290bp, Mid March at approx 270bp

iTraxx average spreads, 5y mean = 37bp



Average market implied survival probability

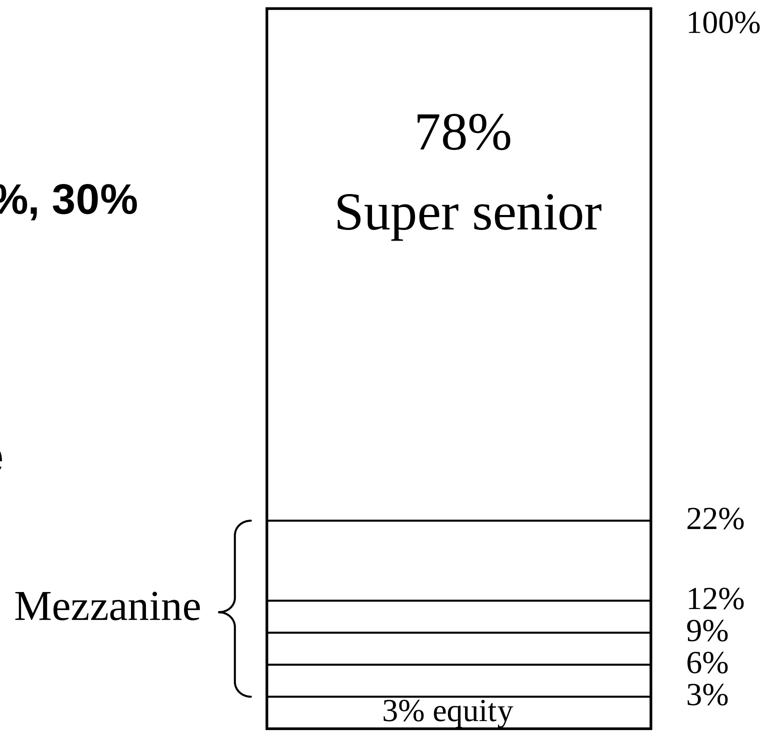


Standardized CDO tranches

- iTraxx Europe 3%, 6%, 9%, 12%, 22%
  - US index CDX has points 3%, 7%, 10%, 15%, 30%
- Has done a lot to provide liquidity in structured credit
- Reliable pricing information available
- Quotation:
  - bp running fee
  - Equity tranche:

    500bp running, quoted on upfront payment!

    Due to timing risk of events



Nordea

Reference Gaussian copula model

- **N credit names, i = 1,…,N**
- **Default times:** $T_i \sim F_i(t) = 1 - \exp\left(-\int_0^t \lambda_i(u)du\right)$
- **λ_i curves bootstrapped from CDS quotes**
- **T_i correlated through the copula:**

  $F_i(T_i) = \Phi(X_i)$ with $X = (X_1,\ldots,X_N)^t \sim N(0,\Sigma)$

  Σ correlation matrix, variance 1, constant correlation ρ

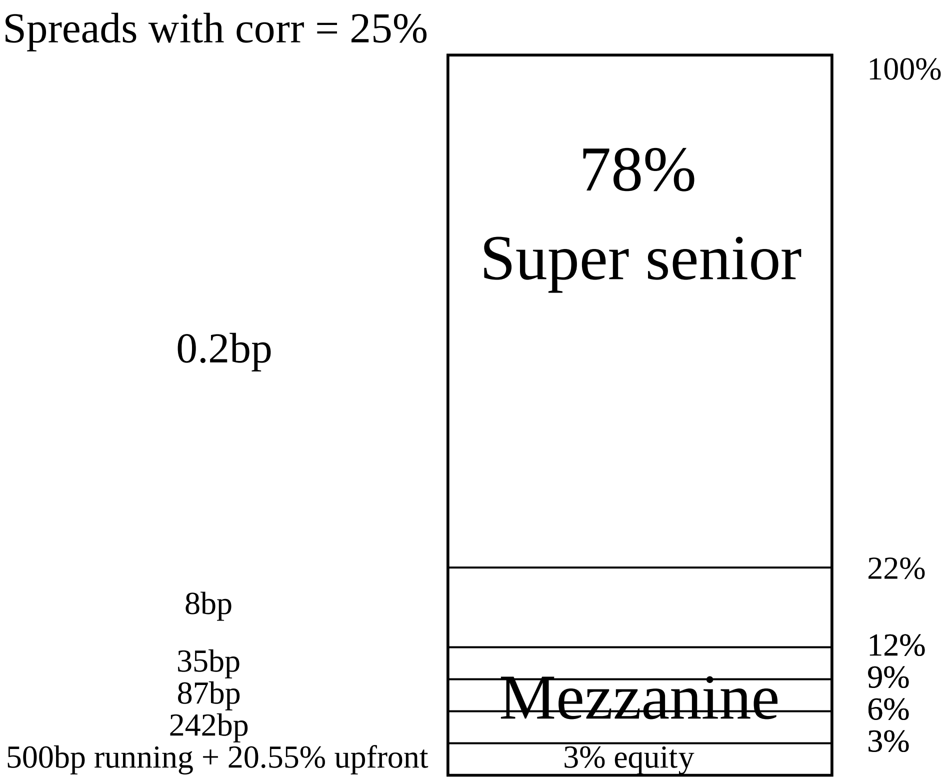
  Could take

$$X_i = \sqrt{\rho}M + \sqrt{1-\rho}Z_i \qquad \Sigma = \begin{pmatrix} 1 & & \\ & \ddots & \rho \\ \rho & \ddots & \\ & & 1 \end{pmatrix}$$

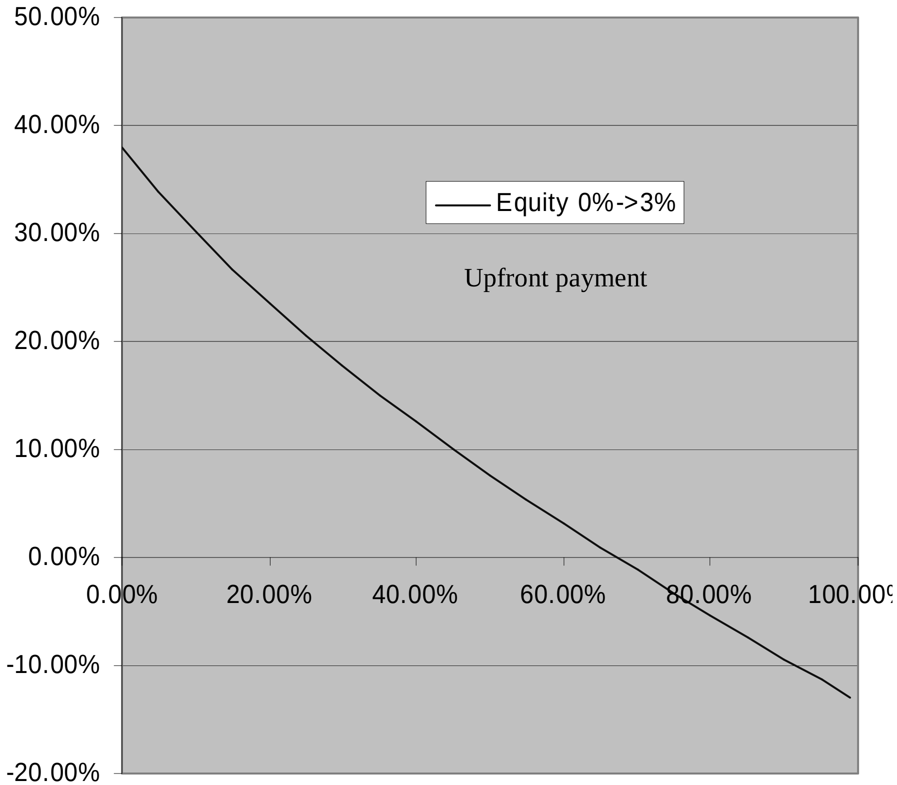
- **In model: correlation independent of product to be priced**

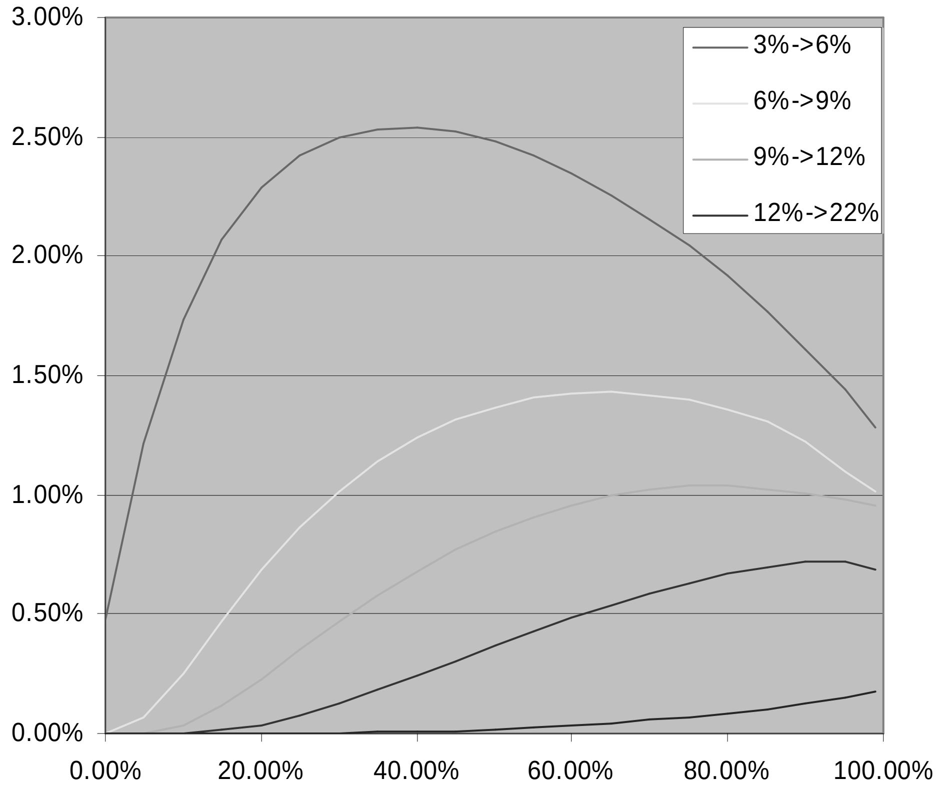
CDO behaviour

- **Structure:**
  - **125 name, iTraxx**
  - **RR almost all 40%**
  - **Avg CDS = 37bp**
  - **Corr = 25%**
  - **Start 11-oct-2005**
  - **5y structure, ends 20-dec-2010**
  - **Premium: 3m, act/360**
  - **Valuation 10-oct-2005**



Correlation dependence
Fair spreads as function of correlation





CDO behaviour depends on

- Number of names
- Spreads of the underlying names
- Tranching:
  - Size of tranche
    - Smaller tranches are more leverage/exposed to changes
  - Order of tranche
- Correlation
- Recovery rate

Prices in the market have a correlation smile

- **In practice:**

  Correlation depends on product, 10-oct-2005, 5Y iTraxx Europe

- **Tranche**
- **Maturity**
- **Fair coupons**

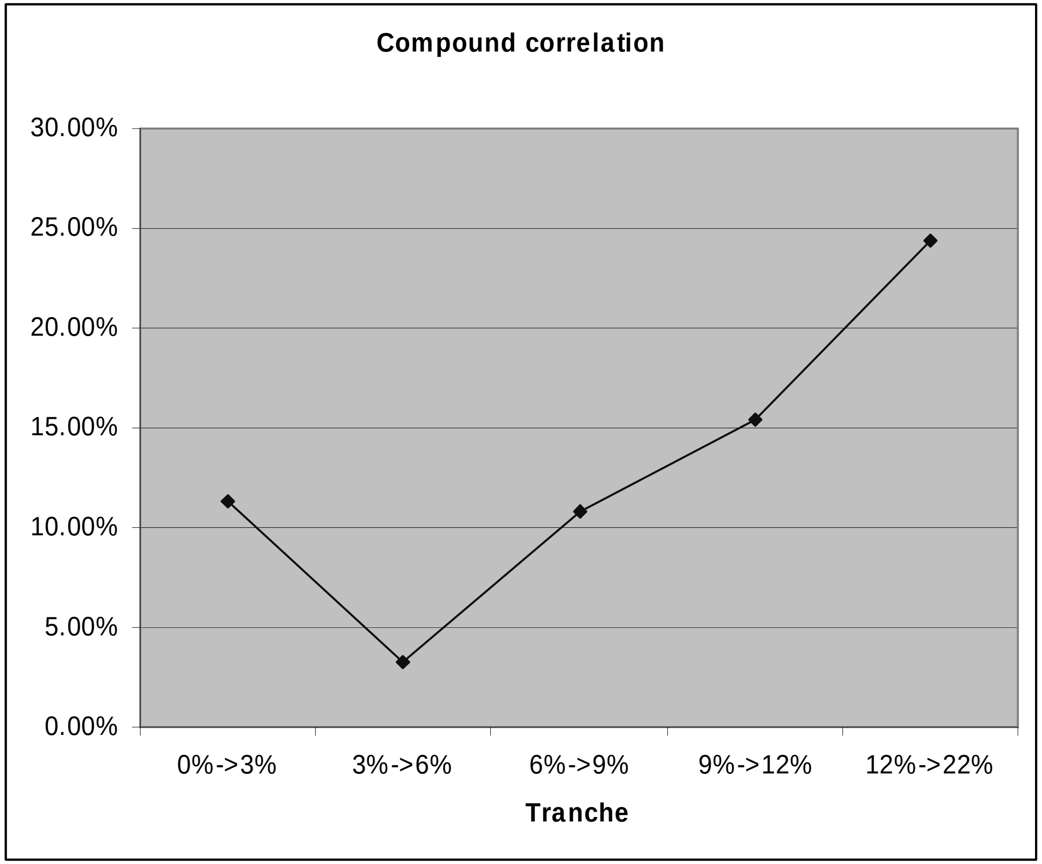
  Equity upfront: 29.2%

  3-6%: 0.97%

  6-9%: 0.28%

  9-12% 0.13%

  12-22%: 0.07%



Compound correlations

- The correlation on the individual tranches
- Mezzanine tranches have low correlation sensitivity and
  even non-unique or non-existent correlation for given spreads!
- No way to extend to, say, 2%-5% tranche
  or bespoke tranches

- What alternatives exists?

Base correlations

Nordea

Steep base correlation curves

Base smile	Corr	Fair coupons
3%	5%	33.94%
6%	30%	-0.65%
9%	45%	-0.13%
12%	50%	0.41%
22%	55%	0.31%

Negative spreads!!

- **Base correlations depends on previous points**
- **Somewhat contradicting the whole idea of base tranches!**

Are base correlations a real solution?

- No, it is merely a convenient way of describing prices on CDO tranches
- An intermediate step towards better models that exhibit a smile
- No general extension to other products
- No smile dynamics
- Interpolation issues
- Correlation smile modelling, versus
- Models with a smile and correlation dynamics
- Base correlation is NOT a model!!
- Nevertheless: they are used a lot!

Why have models?
How to use them?

- We do see prices on the standard tranches in the market, so why have a model at all?
- Interpolation
  - Non standard tranches, e.g. 2%-4%
- Extrapolation
  - Attachment/detachment points below 3%
  - Bespoke portfolios
  - Other products: CDS -> CDO, CDO -> CDO^2, etc.
- Usually: map expected losses to find corr for other tranches
- Risk numbers

Nordea

Delta hedges

- **Delta risk: how much does the NPV change when the underlying credit spreads widen by 1bp?**
- **CDO tranches typical traded with initial credit hedge, i.e. only correlation risk left!**
- **Conveniently quoted as amount of underlying index CDS to buy in order to hedge credit risk, i.e. deltaCDO/deltaCDS**
- **Split out on individual names or just consider index?**
- **Base correlation: find by long/short strategy in the same way as NPV!**

Nordea

Deltas in different models

- **Deltas differ between models:**

Tranche	0%-3%	3%-6%	6%-9%	9%-12%	12%-22%
Compound corr	22.1	9.1	2.7	1.2	0.6
Base corr	22.1	6.1	2.0	0.9	0.5
RFL	25.9	7.5	0.4	0.1	0.1

- **Agreement on delta amounts requires model agreement**
- **Non-unique deltas when spread & correlation is connected, i.e. in models with smile dynamics**

Last 20 schedule and date roll convention

- **End date will be 20th of Mar, Jun, Sep or Dec.**
- **If we have passed any of these dates we roll to the next date, so e.g. end date 21st Jun will roll to 20st Sep, etc.**
- **First period will be long if we would otherwise get less than 1m to first date in scheudle!**
- **Stub/long period in the beginning.**
- **Intermediate points are rolled Following.**
- **Usually in the credit market start and end dates can fall on non-business days.**
- **Always start protection the day after the trade day, even if a non business day.**

Risk ladders

- CDS curve most often bootstrapped from yearly quotes
  - Risk on the yearly quotes, 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, etc.

- However: trades end every quarter
- Risk might move around when crossing 20 Mar, Jun, Sep, Dec
- Example:

Date	4Y	4.25Y	4.50Y	4.75Y	5Y
QTR before roll			200		
Y before roll	100				100
QTR after roll		200			
Y after roll	150				50

- Get risk on a quarterly ladder, even though the curve is still bootstrapped from yearly quotes. Be aware how your risk changes on rolls.

Implementation strategies

- **Key: efficiency, flexibility and fast + accurate risk!**
- **Copula type models:**
  - **Monte Carlo**
  - **Recursive/FFT techniques**

Nordea

Implementation of Gaussian copula by MC

- **Monte Carlo simulation of $X \sim N(0,\Sigma)$**
  - Simulate $Y \sim N(0,I)$
  - Find A such that $AA' = \Sigma$
  - $X = AY$
- **How to find A?**
  - Cholesky decomposition
  - Eigenvalue decomposition: $A = P\text{sqrt}(\lambda)$
  - The latter is better, in particular with Sobol sequences
- **Simulation:**
  - Simulate default time $T_i = F_i^{-1}(\Phi(X_i))$ for all names, and price.
  - Do it, say, 10000 times.
  - Can price any derivative, simple.

Risk numbers in MC pricing

- **"Naive" risk numbers:** $\frac{\partial V}{\partial \lambda_i} = \frac{V(\lambda_i + \varepsilon) - V(\lambda_i)}{\varepsilon}$
- **For credit risk we can exchange differentiation and integration:**

$$V = E[g(\tau)] = \int g(\tau) f(\tau \mid \lambda_1, \ldots, \lambda_N) d\tau$$

$$\begin{aligned}
\frac{\partial}{\partial \varepsilon} V(\lambda_1, \ldots, \lambda_i + \varepsilon s, \ldots, \lambda_N)|_{\varepsilon=0} &= \int g(\tau) \frac{\partial}{\partial \varepsilon} f(\tau \mid \lambda_1, \ldots, \lambda_i + \varepsilon s, \ldots, \lambda_N)|_{\varepsilon=0}\, d\tau \\
&= \int g(\tau) \frac{\partial}{\partial \varepsilon} \log f(\tau \mid \lambda_1, \ldots, \lambda_i + \varepsilon s, \ldots, \lambda_N)|_{\varepsilon=0}\, f(\tau) d\tau \\
&= E[g(\tau) \frac{\partial}{\partial \varepsilon} \log f(\tau \mid \lambda_1, \ldots, \lambda_i + \varepsilon s, \ldots, \lambda_N)|_{\varepsilon=0}]
\end{aligned}$$

- **Calculate derivative and price in same simulation, but for a different payout function**
- **Speed up of factor 5x125=625. Also improves stability!**

Nordea

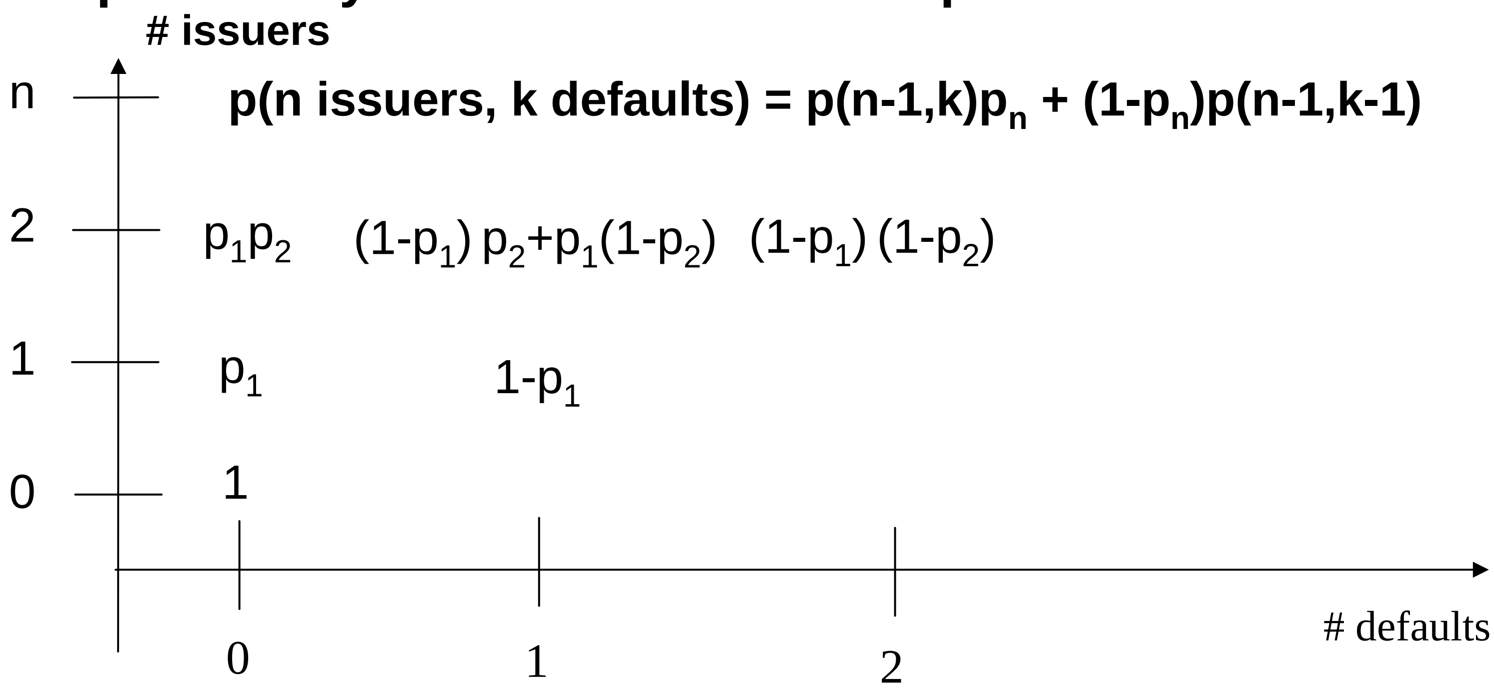
Recursive/FFT implementation

- **Write X~N(0,Σ) as** $X_i = aM + \sqrt{1-a^2} Z_i \qquad a = \sqrt{\rho}$
- **Only viable for derivatives that depend on the number of defaults or the cumulative loss (perhaps discretised if RR or notionals are not equal)**
- **Conditional on M:**
  - **X_i's are independent ⇒**

    For given horizon T, the default indicators $1_{\left(X_i \le C_i(T)\right)}$ **are independent**
  - **Calculate distribution of number of defaults recursively in N = #names**
  - **Binomial expression**
- **Find loss distribution by integrating over M**
- **Fast and no MC noise**

Recursive build-up of loss distribution

- **Conditional on M, given a time horizon t: independence and p_i is the probability name i has survived up to time t.**



- **Next: integrate over M**

What is the survival probability?

- Let $X_i = aM + \sqrt{1-a^2} Z_i$, with $X_i \sim H_i$
- The model matches quantiles: $F_i(T_i) = H_i(X_i)$
- This means the conditional survival probabilities are:

$$
\begin{aligned}
P(T_i \geq t \mid M) &= P(F_i^{-1}(H_i(X_i)) \geq t \mid M) \\
&= P(X_i \geq H_i^{-1}(F_i(t)) \mid M) \\
&= P\left(Z_i \geq \frac{H_i^{-1}(F_i(t)) - aM}{\sqrt{1-a^2}} \mid M\right)
\end{aligned}
$$

The search for better copulas has started...

- **"Better" means**
  - **describing the observed prices in the market for iTraxx**
  - **produces a correlation smile**
  - **has a reasonable low number of parameters**

    one can have a view on and interpret
  - **has a plausible dynamics for the correlation smile**
  - **constant parameters can be used on a range of**
    - **tranches / products**
    - **maturities**
    - **(portfolios)**
  - **Efficient pricing and risk numbers**
- **Often start from Gaussian model described as a 1 factor model**
  - **Computational efficiency!**

$$X_i = aM + \sqrt{1-a^2} Z_i$$

What makes a good practitioner?

- **Has a lot of common sense**
  - **Understands the difference between up and down, elbow and head…**
- **Understands products and markets**
- **IT knowledge**
  - **Excel, Visual Basic, system functionality, …, C++**
  - **Make things operational, streamline repetitive processes, can implement the math so it works, etc.**
- **Mathematical skills**
  - **Required to develop models and make efficient implementation**
  - **Numerical analysis, analysis, algebra, stochastic models, etc.**

Conclusion

- **Models will have to be developed further**
  - **Smile description and dynamics**
  - **Delta amounts and other relevant risk numbers**
  - **Bespoke tranches**
  - **Computational efficiency**
  - **Will have to go through a couple of iterations**
- **Market and products are changing over time**
- **Practical challenges**
  - **Manage market data & information**
  - **Provide smooth infrastructure for all the numbers/trades/etc.**