



## Quantifying counterparty risk

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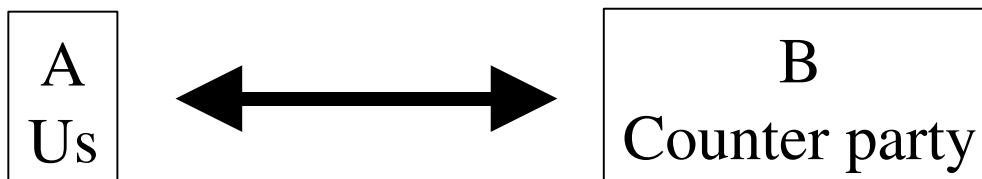
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# Agenda

- **Definitions of counterparty risk**
- **One sided counterparty risk**
- **Product specific considerations**
- **One or two sided counterparty risk?**
- **Model requirements**
- **Trading of counterparty risk**
- **Numerical implementations**
  - 3 suggestions, “regression in MC” in detail
  - How, what to look out for, live demo, ...
- **Portfolio calculations by aggregation**
- **Conclusion**

## Counterparty risk definition

- The risk of losing money on a portfolio of derivative contracts when a counterparty default



- Cashflows at default time  $\tau$  before maturity T:

- Payments before  $\tau$ : according to the contract
- At default of counterparty B:
  - $NPV > 0$ : counterparty owes us money and pays  $RR^{B*} NPV$  to us
  - $NPV < 0$ : we owe the counterparty money and pay them in full
- At our default A:
  - $NPV > 0$ : counterparty owes us money and pay in full
  - $NPV < 0$ : we owe the counterparty money and pay  $RR^{A*} NPV$

# Purpose of measuring counterparty risk

- **Reservations for future exposure**
  - Lines control
- **Pricing**
  - Special price for each counterparty
- **Hedging**
- **Related, but NOT considered here:**
  - VaR, expected shortfall
    - Typical 10 trading days
  - Economic Capital
    - 99.7% quantile of unexpected losses on 1y horizon
- **Accuracy needed**
  - Cash-flows/exposure on individual days or the big picture?

## Other means of managing counterparty risk

- **Netting agreements**
  - Net between contracts with the same counterparty, also across asset classes
  - Almost always in place
- **Collateral agreements**
  - Make sure exposure never exceeds a given threshold by securing the position with collateral
  - Typical for interbank counterparties and large clients
- **Early termination clauses**
- **Corporate counterparties**
  - Smaller portfolios, but no collateral and higher credit risk

## Counterparty risk math definition

$NPV(\tau) = E_{\tau} [CF(\tau, T)]$ , seen from us, counterparty A

payoff<sup>D</sup>(t) =  $1_{\tau > T} CF(t, T) + 1_{t < \tau \leq T} [CF(t, \tau) + df(t, \tau) NPV(\tau) (\gamma^A + \gamma^B)]$

$\gamma^A = 1_{\tau = \tau^A} (RR^A 1_{NPV(\tau) < 0} + 1_{NPV(\tau) > 0})$ , A defaults

$\gamma^B = 1_{\tau = \tau^B} (RR^B 1_{NPV(\tau) > 0} + 1_{NPV(\tau) < 0})$ , B defaults

- This is two sided counterparty risk, both parties can default
- One sided: put  $\gamma^A = 0$  (we cannot default)

## One sided counterparty risk

- $\gamma^A=0$ , we only consider defaults of our counterparty
- With a bit of tedious, but simple, algebra and law of iterated expectations:

$$E_t(\text{payoff}^D(t)) = E_t(\text{payoff}(t)) - (1 - RR^B) E_t \left[ \mathbf{1}_{t < \tau \leq T} df(t, \tau) NPV^+(\tau) \right]$$

Value without counterparty risk

Option part in default case  
Call 0-strike

- RR assumed deterministic
- Adds level of optionality: we need (a function of) the value at a future default date
- Mean over  $\tau$  and NPV values

## Products

- **Bank loan portfolio**
  - Simple --- value of underlying do not change much!
  - Might have extension clause, correlated to credit quality, complicates matters!
- **IRS**
  - Simple
  - Value 0 at initiation, but value  $\neq 0$  at future dates
  - Fast approximations can be made
- **FX**
- **Swaptions**
  - Cash/physical settled makes difference wrt. final maturity
  - Option on option, stochastic volatility
- **Credit products**
  - Take correlation between underlying and counterparty into account
- **Equity**
- **Portfolios of the full monty...**

# IRS: Interest Rate Swaps

- The general expression simplifies:

$$IRS^D(t) = IRS(t) - (1 - RR^B) \int_t^T \text{swaption}(t, s, T, K) dQ(\tau \leq s)$$

- **Q describe default times by hazard rates from CDS quotes**
  - CDS up to 10y, trades up to 30y
- **Independence between  $\tau$  and rates assumed**
  - Rate distribution does not depend on  $\tau$ , i.e. we get vanilla swaption
- **Weighting options with default probabilities**

# Impact on price on a single IRS

- IRS<sup>D</sup> quote: coupon that gives IRS<sup>D</sup>=0
- Market data as of 21-MAR-2007 (rates, vol)
- CDS scenarios:

Tenor	Survival Prob		
	Low CDS 5y=30bp	Medium CDS 5y=100bp	High CDS 5y=300bp
5y	97.50%	91.92%	77.67%
10y	95.07%	84.50%	60.35%
15y	92.71%	77.69%	46.89%
20y	90.40%	71.42%	36.43%

- Results:

Tenor	Maturity Date	Rate	Diff in rates in bp		
			Low CDS 5y=30bp	Medium CDS 5y=100bp	High CDS 5y=300bp
5y	Fri-23-Mar-2012	4.1230%	0.17	0.53	1.50
10y	Thu-23-Mar-2017	4.1890%	0.50	1.62	4.44
15y	Wed-23-Mar-2022	4.2850%	0.91	2.87	7.55
20y	Tue-23-Mar-2027	4.3290%	1.25	3.93	9.96

- Adjustments a bit (times ½) lower than in Brigo & Masetti (2004)

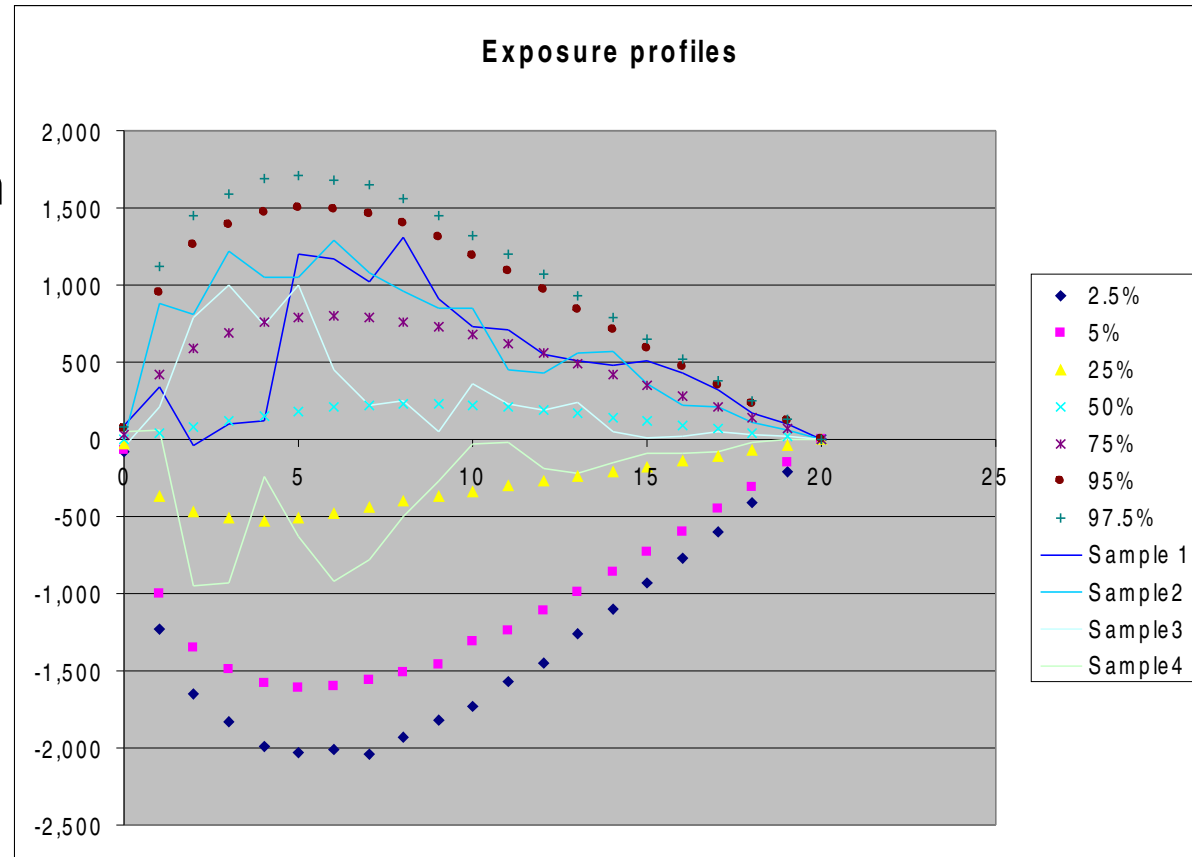
– Vol assumptions different, ...

## One or two sided counterparty risk?

- **Seen from our point of view:**
  - One sided counterparty risk is enough
- **But the counterparty has the same view**
  - So two sided counterparty risk seem to be the way to go if parties should agree on a common price
- **Value depends mostly on difference in CDS spreads**
  - As an approximation only see it from the highest rated counterparty's side

# Exposure profiles

- Jumps at payments dates
- Need to calculate option on full portfolio
  - Cannot do it trade by trade due to netting
  - Exposures occur at different dates for different swaps
- Single trade/portfolio numbers
  - Quantiles, max, quantiles of max, averaging, etc.



## Portfolios of interest rate swaps

- **Netting of positions & exposure**
  - Simple example: payer and receiver swap with same strike and maturity
- **“Swaption” on general cashflow of (libor) payments**
- **Damiano Brigo & Massimo Masetti, 2005 find approximate equations**
  - Either strictly payer or receiver portfolios
  - Both payer and receiver portfolios give complications
    - This will usually be the case!
- **This is going in the direction of specializing for specific products/type of positions/...**
- **In general assuming little about the products or portfolio composition, then more general models must be used...**

# General or specific models: I would say general!

- **Even with specific models there is a limit to what can be handled**
  - Realistic swap portfolio
- **For homogeneous portfolios**
  - Simple regression techniques will be sufficient in order to give good overview
  - Might be rather add-hoc, but never the less be sufficient
  - Per trade: current NPV + add-on
  - Add-on depends on currency (vol?), time to maturity, counterparty rating
  - Give discount in add-on in order to take typical netting into account
- **For non-homogeneous portfolios**
  - Something more general needs to be done anyway
  - In particular for exotics

## Model requirements

- **In general: adds level of optionality**
  - Needs value at a future date  $\tau$  of future remaining payments
- **NPV can depend on history up to default**
  - Simple example: physical settled swaption past expiry date, ITM/OTM?
- **Options**
  - Before expiry: needs to price an option on an option
  - SV models
- **Correlation between default time and underlying**
  - Independence might be reasonable for rates/defaults
  - Credit/equity products: correlation between reference name and counterparty needs to be taken into account
- **The interest is in calculating the option part in the adjusted price**
  - Might use other models than the pricing model as the focus is different

# Trading of counterparty risk

- **So far: pricing taking counterparty risk into account**
  - Used as MTM (seldom) or only in lines surveillance
- **Hedging counterparty risk**
  - Swap, option desks, etc. hedge counterparty risk with credit desk in order to trade more with a given limit
  - Jump To Default risk,  $(1-RR^B)NPV^+$ , current exposure
  - Hazard risk: potential future exposure
- **Make counterparty risk a market risk like delta/vega/...**
- **Difficult to do for smaller names with illiquid CDS market**
- **Risk number calculation adds a lot to numerical problems**
  - Would require a lot more simulations than just the pricing of counterparty risk

# Risk neutral measure $\leftrightarrow$ real world measure

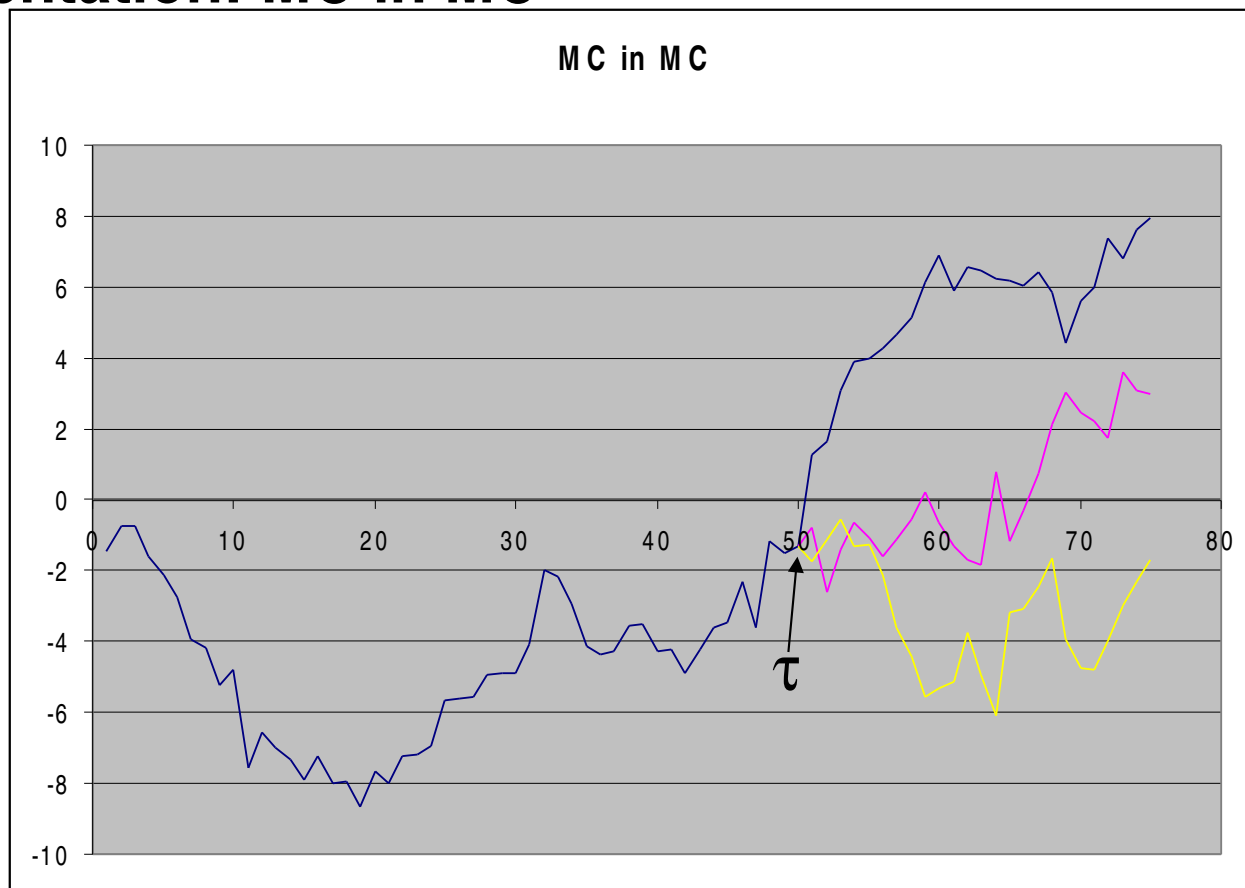
- **Risk neutral measure:**
  - What we have worked with so far
  - Used for pricing and hedging
- **Real world measure:**
  - Risk management might argue that this is more relevant for lines, reservations, etc.
  - Both for market factors and default risk
  - Different models

## Numerical implementation: MC on Grid

- **Original idea by Jesper Andreasen**
- **Suitable when both Grid and MC models available**
  - And products can be priced in grid
- **Do grid once backwards**
  - Store value for every grid point
- **Simulate MC state variables AND defaults forward**
  - Pick a grid box based on default time and state
    - The value of future payments are pre computed from the grid!
  - Allows for default/state variable correlation
- **Haven't tried it....**
- **Another idea: Do grid for default state as well, increases dimensionality, but only 2 states in new direction**

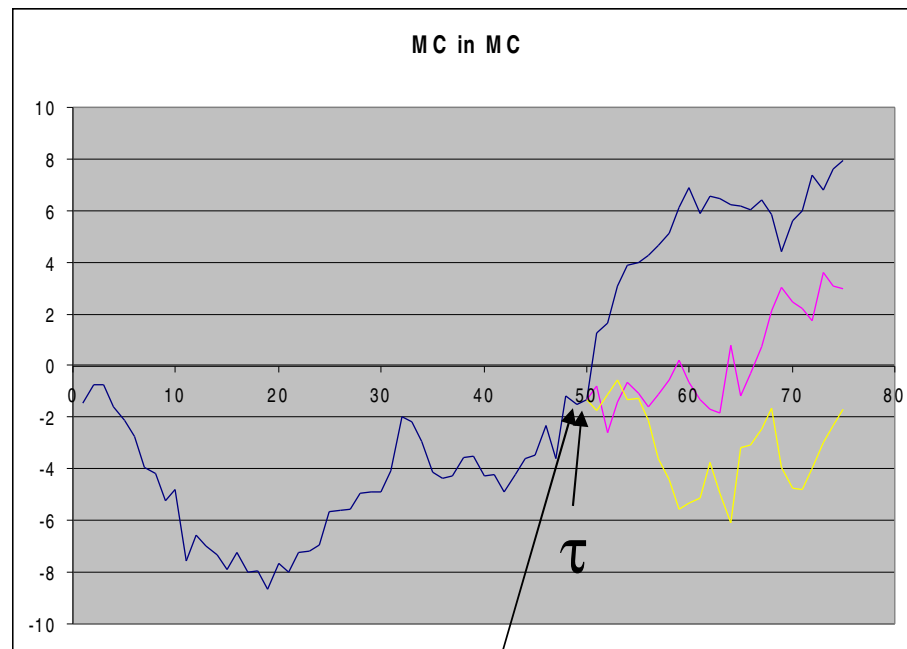
# Numerical implementation: MC in MC

- Procedure:
  - Simulate  $\tau$
  - Value future CF by MC from that point
- Optimizations
  - Product dependent
  - Path in time
  - Jump to date
- Cross asset portfolios/hybrids/...
  - Huge MC engine
- Most exotics are in MC models these days...
- MC in MC explodes computationally,  $\#sim^2$



# Numerical implementation: Regression in MC

- Procedure:
  - Simulate  $\tau$
  - Value future CF by regression at  $\tau$
- Like Longstaff-Schwartz regression for early exercise boundary
- Feasible computationally:
  - 2 x #sim (or less)
  - Perhaps already doing the sim for early exercise boundary



Do regression instead of MC

$$E_t(\text{payoff}^D(t)) = E_t(\text{payoff}(t)) - (1 - RR^B) E_t \left[ 1_{t < \tau \leq T} df(t, \tau) NPV^+(\tau) \right]$$

## Regression in MC

- At each time  $t$ , predict value of future cashflow by regression:

$$NPV(\tau) = \alpha(\tau)'x(\tau) + \varepsilon = \sum_k \alpha_k(\tau)x_k(\tau) + \varepsilon$$

- $NPV(\tau)$ : value of future cashflows at time  $\tau$ , see next slide on how to get
  - Note: NOT  $NPV^+(\tau)$ , as this would make the regression fit worse.
  - Take positive part after the regression!
- $\alpha(\tau)$ : linear regression coefficients at time  $\tau$
- $x(\tau)$ : regression variables like  $\text{libor}$ ,  $\text{swaprate}$ ,  $\text{swaprate}^2$ , etc.
  - Choose with care!
  - Should predict value by just knowing current state of the world
- $\varepsilon(\tau)$ : “noise” vector

# Regression in MC procedure

- **Make pre simulation**
  - Store a set of full paths
  - Evaluate forward in time as usual, store values for each time step
  - Now go backwards in time in order to find value of future CF at each time
  - Find regression coefficients from regression variables
- **Make simulation in model:**
  - Simulate defaults times, either given externally from “credit model”, or given by the model itself when correlation between default and asset needed.
  - Simulate underlyings, rate, etc., as usual
  - Evaluate at time  $t$  forward in time as usual, but for counter party risk:
    - Return 0 if not defaulted, i.e.  $t < \tau$
    - $(\alpha'x)(t)$  if defaulted here, i.e.  $t = \tau$
    - Pass on current value if previously defaulted,  $t > \tau$  (can in some cases be disregarded)

## Regression in MC, considerations

- **Regression variables:**
  - Should predict value of remaining cash-flow from current state of the world
  - Can be a bit tricky to find the best
    - Experiment!
    - Both short end and long end of curve
    - Value of vol with SV models
    - Use powers of variables
  - Need more experience for exotic stuff
- **Regress on full range of values instead of a lot of zeros and the positive part, i.e.  $NPV(\tau)$  instead of  $NPV^+(\tau)$ .**
  - Better fit at fitting stage
  - Better prediction at prediction state
  - Makes aggregation across trades possible at a later stage!

## Live demo.....

- **Implemented as “aggregate model”:**
  - All models can interact with the default model (if they adhere to the interface!)
  - If correlation default  $\leftrightarrow$  asset needed the model can provide default times itself.
- **Implemented with new keyword in trade description to get regression variable**
  - Means pricing and counterparty risk can be done simultaneously!
- **Still lots of rough edges! Work in progress!**
- **This stuff actually works 😊**
  - Give values in line with “closed form” solution for swaps
  - Reasonable performance
  - Low overhead compared to usual pricing (at least for exotics...)

# Future directions

- **Implement risk**
  - Should be an easy extension
  - Credit risk part by standard trick of swapping differentiation and MC mean (integration)
- **Implement the counterparty interface on all models**
- **Implement plumbing to value a whole portfolio of trades in one go**
  - “Super model” to value all assets
  - Might NOT be needed if the same defaults  $\tau$  are used in all models and models return  $NPV(\tau)$  as a vector for all default times.
  - Possible to aggregate information from several independent trades/models

$$NPV_{total}^+(\tau) = \left[ \sum_{\text{trades}} NPV_{trade}(\tau) \right]^+$$

- More accurate regression because tailored to each individual trade
- Simple to aggregate. Store values from, say, EOD, so effect of new trades can easily be calculated

## Conclusion

- **Counterparty risk adds level of optionality**
- **Netting agreements → we should look at a portfolio level**
  - Might be distributed across books at different trading desks
  - A challenge to infrastructure and systems
- **Need to decide on strategy**
  - Get efficient approximations for simple single asset class/product portfolios
  - Do all products/asset classes together in huge MC engine
  - Some route in between or combination...
  - Computations could be challenging!
- **Pricing of counterparty risk can be obtained in roughly the same time as an MC price.**
  - Good enough as probably most interesting for exotics anyway

## References

- Michael Pykhtin (ed.), **Counterparty Credit Risk Modelling --- Risk Management, Pricing and Regulation**, Risk Books, 2005.
- Darrel Duffie and Ming Huang, **Swap Rates and Credit Quality**, Papers and Proceedings of the Fifty-Sixth Annual Meeting of the American Finance Association, San Francisco, California, January 5—7, *The Journal of Finance*, 1996, vol 51 (3), pp. 921-949.
- Damiano Brigo and Massimo Masetti, **Chapter 10: Risk Neutral Pricing of Counterparty Risk**, in Michael Pykhtin (ed.), **Counterparty Credit Risk Modelling --- Risk Management, Pricing and Regulation**, Risk Books, 2005.
- Damiano Brigo and Andrea Pallavicini, **Counterparty risk valuation under correlation between interest-rates and default**, *Credit Models --- Banca IMI*, 14 Dec 2006.