Quantifying counterparty risk

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Agenda

- Definitions of counterparty risk
- One sided counterparty risk
- Product specific considerations
- One or two sided counterparty risk?
- Model requirements
- Trading of counterparty risk
- Numerical implementations
  - 3 suggestions, “regression in MC” in detail
  - How, what to look out for, live demo, ...
- Portfolio calculations by aggregation
- Conclusion
Counterparty risk definition

- The risk of losing money on a portfolio of derivative contracts when a counterparty default

\[ \begin{align*}
A & \hspace{0.5cm} Us & B \\
\text{Counter party} & & \\
\end{align*} \]

- Cashflows at default time \( \tau \) before maturity \( T \):
  - Payments before \( \tau \): according to the contract
  - At default of counterparty B:
    - \( \text{NPV} > 0 \): counterparty owes us money and pays \( RR^B \times \text{NPV} \) to us
    - \( \text{NPV} < 0 \): we owe the counterparty money and pay them in full
  - At our default A:
    - \( \text{NPV} > 0 \): counterparty owes us money and pay in full
    - \( \text{NPV} < 0 \): we owe the counterparty money and pay \( RR^A \times \text{NPV} \)
Purpose of measuring counterparty risk

- Reservations for future exposure
  - Lines control

- Pricing
  - Special price for each counterparty

- Hedging

- Related, but NOT considered here:
  - VaR, expected shortfall
    - Typical 10 trading days
  - Economic Capital
    - 99.7% quantile of unexpected losses on 1y horizon

- Accuracy needed
  - Cash-flows/exposure on individual days or the big picture?
Other means of managing counterparty risk

• Netting agreements
  – Net between contracts with the same counterparty, also across asset classes
  – Almost always in place

• Collateral agreements
  – Make sure exposure never exceeds a given threshold by securing the position with collateral
  – Typical for interbank counterparties and large clients

• Early termination clauses

• Corporate counterparties
  – Smaller portfolios, but no collateral and higher credit risk
Counterparty risk math definition

\[ NPV(\tau) = E_\tau [ CF(\tau, T) ] \], seen from us, counterparty A

payoff_D(t) = 1_{\tau > T} CF(t, T) + 1_{t < \tau \leq T} \left[ CF(t, \tau) + df(t, \tau) NPV(\tau)(\gamma^A + \gamma^B) \right]

\[ \gamma^A = 1_{\tau = \tau^A} \left( RR^A 1_{NPV(\tau) < 0} + 1_{NPV(\tau) > 0} \right), \text{ A defaults} \]

\[ \gamma^B = 1_{\tau = \tau^B} \left( RR^B 1_{NPV(\tau) > 0} + 1_{NPV(\tau) < 0} \right), \text{ B defaults} \]

• This is two sided counterparty risk, both parties can default
• One sided: put \( \gamma^A = 0 \) (we cannot default)
One sided counterparty risk

- $\gamma^A=0$, we only consider defaults of our counterparty
- With a bit of tedious, but simple, algebra and law of iterated expectations:

$$E_t(\text{payoff}^D(t)) = E_t(\text{payoff}(t)) - (1 - RR^B) E_t[1_{t<\tau \leq T} df(t, \tau) \text{NPV}^+(\tau)]$$

Value without counterparty risk

Option part in default case

Call 0-strike

- RR assumed deterministic
- Adds level of optionality: we need (a function of) the value at a future default date
- Mean over $\tau$ and NPV values
Products

- Bank loan portfolio
  - Simple --- value of underlying do not change much!
  - Might have extension clause, correlated to credit quality, complicates matters!

- IRS
  - Simple
  - Value 0 at initiation, but value \( \neq 0 \) at future dates
  - Fast approximations can be made

- FX

- Swaptions
  - Cash/physical settled makes difference wrt. final maturity
  - Option on option, stochastic volatility

- Credit products
  - Take correlation between underlying and counterparty into account

- Equity

- Portfolios of the full monty...
IRS: Interest Rate Swaps

• The general expression simplifies:

\[ IRS^D(t) = IRS(t) - (1 - RR^B) \int_t^T \text{swaption}(t, s, T, K) dQ(\tau \leq s) \]

• Q describe default times by hazard rates from CDS quotes
  – CDS up to 10y, trades up to 30y

• Independence between \( \tau \) and rates assumed
  – Rate distribution does not depend on \( \tau \), i.e. we get vanilla swaption

• Weighting options with default probabilities
Impact on price on a single IRS

- \( \text{IRS}^D \) quote: coupon that gives \( \text{IRS}^D = 0 \)
- Market data as of 21-MAR-2007 (rates, vol)
- CDS scenarios:

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Survival Prob</th>
<th>Low CDS 5y=30bp</th>
<th>Medium CDS 5y=100bp</th>
<th>High CDS 5y=300bp</th>
</tr>
</thead>
<tbody>
<tr>
<td>5y</td>
<td></td>
<td>97.50%</td>
<td>91.92%</td>
<td>77.67%</td>
</tr>
<tr>
<td>10y</td>
<td></td>
<td>95.07%</td>
<td>84.50%</td>
<td>60.35%</td>
</tr>
<tr>
<td>15y</td>
<td></td>
<td>92.71%</td>
<td>77.69%</td>
<td>46.89%</td>
</tr>
<tr>
<td>20y</td>
<td></td>
<td>90.40%</td>
<td>71.42%</td>
<td>36.43%</td>
</tr>
</tbody>
</table>

- **Results:**

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Maturity Date</th>
<th>Rate</th>
<th>Diff in rates in bp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Low CDS 5y=30bp</td>
<td>Medium CDS 5y=100bp</td>
</tr>
<tr>
<td>5y</td>
<td>Fri-23-Mar-2012</td>
<td>4.1230%</td>
<td>0.17</td>
</tr>
<tr>
<td>10y</td>
<td>Thu-23-Mar-2017</td>
<td>4.1890%</td>
<td>0.50</td>
</tr>
<tr>
<td>15y</td>
<td>Wed-23-Mar-2022</td>
<td>4.2850%</td>
<td>0.91</td>
</tr>
<tr>
<td>20y</td>
<td>Tue-23-Mar-2027</td>
<td>4.3290%</td>
<td>1.25</td>
</tr>
</tbody>
</table>

- **Adjustments a bit (times \( \frac{1}{2} \)) lower than in Brigo & Masetti (2004)**
  - Vol assumptions different, ...
One or two sided counterparty risk?

- **Seen from our point of view:**
  - One sided counterparty risk is enough

- **But the counterparty has the same view**
  - So two sided counterparty risk seem to be the way to go if parties should agree on a common price

- **Value depends mostly on difference in CDS spreads**
  - As an approximation only see it from the highest rated counterparty’s side
Exposure profiles

- Jumps at payments dates
- Need to calculate option on full portfolio
  - Cannot do it trade by trade due to netting
  - Exposures occur at different dates for different swaps
- Single trade/portfolio numbers
  - Quantiles, max, quantiles of max, averaging, etc.
Portfolios of interest rate swaps

- Netting of positions & exposure
  - Simple example: payer and receiver swap with same strike and maturity

- “Swaption” on general cashflow of (libor) payments

- Damiano Brigo & Massimo Masetti, 2005 find approximate equations
  - Either strictly payer or receiver portfolios
  - Both payer and receiver portfolios give complications
    - This will usually be the case!

- This is going in the direction of specializing for specific products/type of positions/...

- In general assuming little about the products or portfolio composition, then more general models must be used...
General or specific models: I would say general!

- Even with specific models there is a limit to what can be handled
  - Realistic swap portfolio

- For homogeneous portfolios
  - Simple regression techniques will be sufficient in order to give good overview
  - Might be rather add-hoc, but never the less be sufficient
  - Per trade: current NPV + add-on
  - Add-on depends on currency (vol?), time to maturity, counterparty rating
  - Give discount in add-on in order to take typical netting into account

- For non-homogeneous portfolios
  - Something more general needs to be done anyway
  - In particular for exotics
Model requirements

- In general: adds level of optionality
  - Needs value at a future date $\tau$ of future remaining payments

- NPV can depend on history up to default
  - Simple example: physical settled swaption past expiry date, ITM/OTM?

- Options
  - Before expiry: needs to price an option on an option
  - SV models

- Correlation between default time and underlying
  - Independence might be reasonable for rates/defaults
  - Credit/equity products: correlation between reference name and counterparty needs to be taken into account

- The interest is in calculating the option part in the adjusted price
  - Might use other models than the pricing model as the focus is different
Trading of counterparty risk

- So far: pricing taking counterparty risk into account
  - Used as MTM (seldom) or only in lines surveillance

- Hedging counterparty risk
  - Swap, option desks, etc. hedge counterparty risk with credit desk in order to trade more with a given limit
  - Jump To Default risk, \((1-RR^8)\text{NPV}^+\), current exposure
  - Hazard risk: potential future exposure

- Make counterparty risk a market risk like delta/vega/…

- Difficult to do for smaller names with illiquid CDS market

- Risk number calculation adds a lot to numerical problems
  - Would require a lot more simulations than just the pricing of counterparty risk
Risk neutral measure ↔ real world measure

- **Risk neutral measure:**
  - What we have worked with so far
  - Used for pricing and hedging

- **Real world measure:**
  - Risk management might argue that this is more relevant for lines, reservations, etc.
  - Both for market factors and default risk
  - Different models
Numerical implementation: MC on Grid

• Original idea by Jesper Andreasen

• Suitable when both Grid and MC models available
  – And products can be priced in grid

• Do grid once backwards
  – Store value for every grid point

• Simulate MC state variables AND defaults forward
  – Pick a grid box based on default time and state
    – The value of future payments are pre computed from the grid!
    – Allows for default/state variable correlation

• Haven’t tried it…..

• Another idea: Do grid for default state as well, increases dimensionality, but only 2 states in new direction
Numerical implementation: MC in MC

- Procedure:
  - Simulate $\tau$
  - Value future CF by MC from that point

- Optimizations
  - Product dependent
  - Path in time
  - Jump to date

- Cross asset portfolios/hybrids/…
  - Huge MC engine

- Most exotics are in MC models these days…

- MC in MC explodes computationally, $\#\text{sim}^2$
Numerical implementation: Regression in MC

- **Procedure:**
  - Simulate $\tau$
  - Value future CF by regression at $\tau$

- **Like Longstaff-Schwartz regression for early exercise boundary**

- **Feasible computationally:**
  - $2 \times \#\text{sim}$ (or less)
  - Perhaps already doing the sim for early exercise boundary

\[
E_t(\text{payoff}^D(t)) = E_t(\text{payoff}(t)) - (1 - RR^B)E_t[1_{t < \tau \leq T}df(t, \tau)\text{NPV}^+(\tau)]
\]
Regression in MC

- At each time t, predict value of future cashflow by regression:

\[ NPV(\tau) = \alpha(\tau)' x(\tau) + \varepsilon = \sum_k \alpha_k(\tau) x_k(\tau) + \varepsilon \]

- \( NPV(\tau) \): value of future cashflows at time \( \tau \), see next slide on how to get
  - Note: NOT \( NPV^{+}(\tau) \), as this would make the regression fit worse.
  - Take positive part after the regression!

- \( \alpha(\tau) \): linear regression coefficients at time \( \tau \)

- \( x(\tau) \): regression variables like libor, swaprate, \( \text{swaprate}^2 \), etc.
  - Choose with care!
  - Should predict value by just knowing current state of the world

- \( \varepsilon(\tau) \): “noise” vector
Regression in MC procedure

- Make pre simulation
  - Store a set of full paths
  - Evaluate forward in time as usual, store values for each time step
  - Now go backwards in time in order to find value of future CF at each time
  - Find regression coefficients from regression variables

- Make simulation in model:
  - Simulate defaults times, either given externally from “credit model”, or given by the model itself when correlation between default and asset needed.
  - Simulate underlyings, rate, etc., as usual
  - Evaluate at time t forward in time as usual, but for counter party risk:
    - Return 0 if not defaulted, i.e. t<\(\tau\)
    - \((\alpha'x)(t)\) if defaulted here, i.e. t=\(\tau\)
    - Pass on current value if previously defaulted, t>\(\tau\) (can in some cases be disregarded)
Regression in MC, considerations

• Regression variables:
  – Should predict value of remaining cash-flow from current state of the world
  – Can be a bit tricky to find the best
    – Experiment!
    – Both short end and long end of curve
    – Value of vol with SV models
    – Use powers of variables
  – Need more experience for exotic stuff

• Regress on full range of values instead of a lot of zeros and the positive part, i.e. NPV(\(\tau\)) instead of NPV^+(\(\tau\)).
  – Better fit at fitting stage
  – Better prediction at prediction state
  – Makes aggregation across trades possible at a later stage!
Live demo.....

• Implemented as “aggregate model”:
  – All models can interact with the default model (if they adhere to the interface!)
  – If correlation default ↔ asset needed the model can provide default times itself.

• Implemented with new keyword in trade description to get regression variable
  – Means pricing and counterparty risk can be done simultaneously!

• Still lots of rough edges! Work in progress!

• This stuff actually works 😊
  – Give values in line with “closed form” solution for swaps
  – Reasonable performance
  – Low overhead compared to usual pricing (at least for exotics...)
Future directions

- **Implement risk**
  - Should be an easy extension
  - Credit risk part by standard trick of swapping differentiation and MC mean (integration)

- **Implement the counterparty interface on all models**

- **Implement plumbing to value a whole portfolio of trades in one go**
  - “Super model” to value all assets
  - Might NOT be needed if the same defaults $\tau$ are used in all models and models return $\text{NPV}(\tau)$ as a vector for all default times.
  - Possible to aggregate information from several independent trades/models

\[
\text{NPV}_{\text{total}}^+ (\tau) = \left[ \sum_{\text{trades}} \text{NPV}_{\text{trade}} (\tau) \right]^+
\]

  - More accurate regression because tailored to each individual trade
  - Simple to aggregate. Store values from, say, EOD, so effect of new trades can easily be calculated
Conclusion

- Counterparty risk adds level of optionality

- Netting agreements → we should look at a portfolio level
  - Might be distributed across books at different trading desks
  - A challenge to infrastructure and systems

- Need to decide on strategy
  - Get efficient approximations for simple single asset class/product portfolios
  - Do all products/asset classes together in huge MC engine
  - Some route in between or combination...
  - Computations could be challenging!

- Pricing of counterparty risk can be obtained in roughly the same time as an MC price.
  - Good enough as probably most interesting for exotics anyway
References


- Damiano Brigo and Andrea Pallavicini, Counterparty risk valuation under correlation between interest-rates and default, Credit Models --- Banca IMI, 14 Dec 2006.