Quantifying counterparty risk

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Agenda

- Definitions of counterparty risk
- One sided counterparty risk
- Product specific considerations
  - Interest rate swaps
- One or two sided counterparty risk?
- Model requirements
- Numerical implementations
- Trading of counterparty risk
- Conclusion
Counterparty risk definition

- The risk of losing money on a portfolio of derivative contracts when a counterparty defaults

- Cashflows at default time $\tau$ before maturity $T$:
  - Payments before $\tau$: according to the contract
  - At default of counterparty B:
    - $\text{NPV}>0$: counterparty owes us money and pays $\text{RR}^B \times \text{NPV}$ to us
    - $\text{NPV}<0$: we owe the counterparty money and pay them in full
  - At our default A:
    - $\text{NPV}>0$: counterparty owes us money and pay in full
    - $\text{NPV}<0$: we owe the counterparty money and pay $\text{RR}^A \times \text{NPV}$
Purpose of measuring counterparty risk

- Reservations for future exposure
  - Lines control

- Pricing
  - Special price for each counterparty

- Hedging

- Related, but NOT considered here:
  - VaR, expected shortfall
    - Typical 10 trading days
  - Economic Capital
    - 99.7% quantile of unexpected losses on 1y horizon
Other means of managing counterparty risk

- **Netting agreements**
  - Net between contracts with the same counterparty, also across asset classes
  - Almost always in place

- **Collateral agreements**
  - Make sure exposure never exceeds a given threshold by securing the position with collateral
  - Typical for interbank counterparties and large clients

- **Early termination clauses**

- **Corporate counterparties**
  - Smaller portfolios, but no collateral and higher credit risk
Counterparty risk math definition

\[ NPV(\tau) = E_\tau [CF(\tau, T)] \]

payoff^D(t) = 1_{\tau>T} CF(t, T) + 1_{t<\tau\leq T} [CF(t, \tau) + df(t, \tau)NPV(\tau)(\gamma^A + \gamma^B)]

\[ \gamma^A = 1_{\tau=\tau^A} \left( RR^A 1_{NPV(\tau)<0} + 1_{NPV(\tau)>0} \right) \]

\[ \gamma^B = 1_{\tau=\tau^B} \left( RR^B 1_{NPV(\tau)>0} + 1_{NPV(\tau)<0} \right) \]

• This is two sided counterparty risk, both parties can default
• One sided: put \gamma^A=0 (we cannot default)
One sided counterparty risk

- $\gamma^A = 0$, we only consider defaults of our counterparty
- With a bit of tedious, but simple, algebra and law of iterated expectations:

$$E_t (\text{payoff}^D(t)) = E_t (\text{payoff}(t)) - (1 - RR^B) E_t \left[ \int_{t < \tau \leq T} df(t, \tau) NPV^+(\tau) \right]$$

Value without counterparty risk

Option part in default case
Call 0-strike

- RR assumed deterministic
- Adds level of optionality: we need (a function of) the value at a future default date
- Mean over $\tau$ and NPV values
Products

- Bank loan portfolio
  - Simple --- value of underlying do not change much!

- IRS
  - Simple
  - Value 0 at initiation, but value $\neq 0$ at future dates
  - Fast approximations can be made

- FX

- Swaptions
  - Cash/physical settled makes difference wrt. final maturity
  - Option on option, stochastic volatility

- Credit products
  - Take correlation between underlying and counterparty into account

- Equity

- Portfolios of the full monty...
IRS: Interest Rate Swaps

- The general expression simplifies:

\[ IRS^D(t) = IRS(t) - (1 - RR^B) \int_{t}^{T} \text{swaption}(t, s, T, K) dQ(\tau \leq s) \]

- \( Q \) describe default times by hazard rates from CDS quotes
  - CDS up to 10y, trades up to 30y
- Independence between \( \tau \) and rates assumed
  - Rate distribution does not depend on \( \tau \), i.e. we get vanilla swaption
- Weighting options with default probabilities
Impact on price on a single IRS

- **IRS$^D$ quote**: coupon that gives IRS$^D=0$
- **Market data as of 21-MAR-2007 (rates, vol)**
- **CDS scenarios:**

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Survival Prob</th>
<th>Low CDS 5y=30bp</th>
<th>Medium CDS 5y=100bp</th>
<th>High CDS 5y=300bp</th>
</tr>
</thead>
<tbody>
<tr>
<td>5y</td>
<td>97.50%</td>
<td>91.92%</td>
<td>77.67%</td>
<td></td>
</tr>
<tr>
<td>10y</td>
<td>95.07%</td>
<td>84.50%</td>
<td>60.35%</td>
<td></td>
</tr>
<tr>
<td>15y</td>
<td>92.71%</td>
<td>77.69%</td>
<td>46.89%</td>
<td></td>
</tr>
<tr>
<td>20y</td>
<td>90.40%</td>
<td>71.42%</td>
<td>36.43%</td>
<td></td>
</tr>
</tbody>
</table>

- **Results:**

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Maturity Date</th>
<th>Rate</th>
<th>Diff in rates in bp</th>
<th>Low CDS 5y=30bp</th>
<th>Medium CDS 5y=100bp</th>
<th>High CDS 5y=300bp</th>
</tr>
</thead>
<tbody>
<tr>
<td>5y</td>
<td>Fri-23-Mar-2012</td>
<td>4.1230%</td>
<td>0.17</td>
<td>0.53</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td>10y</td>
<td>Thu-23-Mar-2017</td>
<td>4.1890%</td>
<td>0.50</td>
<td>1.62</td>
<td>4.44</td>
<td></td>
</tr>
<tr>
<td>15y</td>
<td>Wed-23-Mar-2022</td>
<td>4.2850%</td>
<td>0.91</td>
<td>2.87</td>
<td>7.55</td>
<td></td>
</tr>
<tr>
<td>20y</td>
<td>Tue-23-Mar-2027</td>
<td>4.3290%</td>
<td>1.25</td>
<td>3.93</td>
<td>9.96</td>
<td></td>
</tr>
</tbody>
</table>

- **Adjustments a bit (times ½) lower than in Brigo & Masetti (2004)**
  - Vol assumptions different, …
One or two sided counterparty risk?

- **Seen from our point of view:**
  - One sided counterparty risk is enough

- **But the counterparty has the same view**
  - So two sided counterparty risk seem to be the way to go if parties should agree on a common price

- **Value depends mostly on difference in CDS spreads**
  - As an approximation only see it from the highest rated counterparty’s side
Exposure profiles

- Jumps at payments dates
- Need to calculate option on full portfolio
  - Cannot do it trade by trade due to netting
  - Exposures occur at different dates for different swaps
- Single trade/portfolio numbers
  - Quantiles, max, quantiles of max, averaging, etc.
Portfolios of interest rate swaps

- Netting of positions & exposure
  - Simple example: payer and receiver swap with same strike and maturity
- “Swaption” on general cashflow of (libor) payments
- Damiano Brigo & Massimo Masetti, 2005 find approximate equations
  - Either strictly payer or receiver portfolios
  - Both payer and receiver portfolios give complications
    - This will usually be the case!

- This is going in the direction of specializing for specific products/type of positions/…
- In general assuming little about the products or portfolio composition, then more general models must be used…
Model requirements

- In general: adds level of optionality
  - Needs value at a future date $\tau$ of future remaining payments

- NPV can depend on history up to default
  - Simple example: physical settled swaption past expiry date, ITM/OTM?

- Options
  - Before expiry: needs to price an option on an option
  - SV models

- Correlation between default time and underlying
  - Independence might be reasonable for rates/defaults
  - Credit/equity products: correlation between reference name and counterparty needs to be taken into account

- The interest is in calculating the option part in the adjusted price
  - Might use other models than the pricing model as the focus is different
Numerical implementation: MC in MC

- **Procedure:**
  - Simulate $\tau$
  - Value future CF by MC from that point

- **Optimizations**
  - Product dependent
  - Path in time
  - Jump to date

- **Cross asset portfolios/hybrids/...**
  - Huge MC engine
Numerical implementation: MC on Grid

- Original idea by Jesper Andreasen
- Suitable when both Grid and MC models available
  - And products can be priced in grid
- Do grid once backwards
  - Store value for every grid point
- Simulate MC state variables AND defaults forward
  - Pick a grid box based on default time and state
    - The value of future payments are pre computed from the grid!
    - Allows for default/state variable correlation
- Haven’t tried it yet....
- Another idea: Do grid for default state as well, increases dimensionality, but only 2 states
Trading of counterparty risk

- So far: pricing taking counterparty risk into account
  - Used as MTM (seldom) or only in lines surveillance

- Hedging counterparty risk
  - Swap, option desks, etc. hedge counterparty risk with credit desk in order to trade more with a given limit
  - Jump To Default risk, \((1-RR^B)NPV^+\), current exposure
  - Hazard risk: potential future exposure

- Make counterparty risk a market risk like delta/vega/…

- Difficult to do for smaller names with illiquid CDS market

- Risk number calculation adds a lot to numerical problems
  - Would require a lot more simulations than just the pricing of counterparty risk
Risk neutral measure ↔ real world measure

- **Risk neutral measure:**
  - What we have worked with so far
  - Used for pricing and hedging

- **Real world measure:**
  - Some would argue that this is more relevant for lines, reservations, etc.
  - Both for market factors and default risk
  - Different models
Conclusion

- Counterparty risk adds level of optionality
- Netting agreements → we should look at a portfolio level
  - Might be distributed across books at different trading desks
  - A challenge to infrastructure and systems
- Need to decide on strategy
  - Get efficient approximations for simple single asset class/product portfolios
  - Do all products/asset classes together in huge MC engine
  - Some route in between...
  - Computations are going to challenging!
References


- Damiano Brigo and Andrea Pallavicini, Counterparty risk valuation under correlation between interest-rates and default, Credit Models --- Banca IMI, 14 Dec 2006.